Competitive Car Racing with Multiple Vehicles using a Parallelized Optimization with Safety Guarantee

Suiyi He\textsuperscript{1*}, Jun Zeng\textsuperscript{2*}, and Koushil Sreenath\textsuperscript{2}

Abstract—This paper presents a novel planning and control strategy for competing with multiple vehicles in a car racing scenario. The proposed racing strategy switches between two modes. When there are no surrounding vehicles, a learning-based model predictive control (MPC) trajectory planner is used to guarantee that the ego vehicle achieves better lap timing. When the ego vehicle is competing with other surrounding vehicles to overtake, an optimization-based planner generates multiple dynamically-feasible trajectories through parallel computation. Each trajectory is optimized under a MPC formulation with different homotopic Bezier-curve reference paths lying laterally between surrounding vehicles. The time-optimal trajectory among these different homotopic trajectories is selected and a low-level MPC controller with obstacle avoidance constraints is used to guarantee system safety-critical performance. The proposed algorithm has the capability to generate collision-free trajectories and track them while enhancing the lap timing performance with steady low computational complexity, outperforming existing approaches in both timing and performance for a car racing environment. To demonstrate the performance of our racing strategy, we simulate with multiple randomly generated moving vehicles on the track and test the ego vehicle's overtake maneuvers.

I. INTRODUCTION

A. Motivation

Recently, autonomous racing is an active subtopic in the field of autonomous driving research. In autonomous racing, the ego car is required to drive along a specific track with an aggressive behavior, such that it is capable of competing with other agents on the same track. By overtaking other leading vehicles and moving ahead, the ego vehicle can finish the racing competition with a smaller lap time. While the behavior of overtaking other vehicles has been studied in autonomous driving on public roads, however, these techniques are not effective on a race track. This is because autonomous vehicles are guided by dedicated lanes on public roads to succeed in lane follow and lane change behaviors, while the racing vehicles compete in the limited-width tracks without guidance from well-defined lanes. Existing work focuses on a variety of algorithms for autonomous car racing, but most of them could not provide a time-optimal behavior with high update frequency in the presence of other moving agents on the race track. In order to generate racing behaviors for the ego racing car, we propose a racing algorithm for planning and control that enables the ego vehicle to maintain time-optimal maneuvers in the absence of local vehicles, and fast overtake maneuvers when local vehicles exist, as shown in Fig. 1.

B. Related Work

In recent years, researchers have been focusing on planning and control for autonomous driving on public roads. For competitive scenarios like autonomous lane change or lane merge, both model-based methods [1] and learning-based methods [2] have been demonstrated to generate the ego vehicle’s desired trajectory. Similarly, control using model-based methods [3], [4] and learning-based methods [5] have also been developed. However, the criteria to evaluate planning and control performance are different for car racing compared to autonomous driving on public roads. For car racing [6], when the ego racing car competes with other surrounding vehicles, most on-road traffic rules are not effective. Instead of maneuvers that offer a smooth and safe ride, aggressive maneuvers that push the vehicle to its dynamics limit [7] or even beyond its dynamics limit [8] are sought to win the race. In order to quickly overtake surrounding vehicles, overtake maneuvers with tiny distances between the cars and large orientation changes are needed. Moreover, due to the bigger slip angle caused by changing the steering orientation more quickly during racing, more accurate dynamical models should be used for autonomous racing planning and control design. We next enumerate the related work in several specific areas.
increases excessively when multiple vehicles compete with the other vehicle’s strategy and the complexity of the planner competition. In [12], it is assumed that the planner knows These assumptions are relatively simple for a real car racing straight track with one constant-speed surrounding vehicle. In [11], the ego vehicle is assumed to compete on a portions of a specific track [29]. However, these approaches don’t solve all challenges. For instance, work in [13], [18], [28], [29] does not take lap timing enhancement into ac-
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As mentioned above, all the previous work on planning and control design for autonomous racing could not enhance the lap timing performance and simultaneously compete with multiple vehicles. Inspired by the work on iterative learning-based control and optimization-based planning, we propose a novel racing strategy to resolve the challenges mentioned above with a steady low computational complexity.

C. Contribution

The contributions of this paper are as follows:

• We present an autonomous car racing strategy that switches between a learning-based MPC trajectory planner (in the absence of surrounding vehicles) and optimization-based homotopic trajectory planner with a low-level safety-critical controller (when the ego vehicle competes with surrounding vehicles).

• The learning-based MPC approach guarantees time-optimal performance in the absence of surrounding vehicles. When the ego vehicle competes with surrounding vehicles, multiple homotopic trajectories are optimized in parallel with different geometric reference paths and the best time-optimal trajectory is selected to be tracked with an optimization-based controller with obstacle avoidance constraints.

• We validate the robust performance together with steady low computational complexity of our racing strategy in numerical simulations where randomly moving vehicles are generated on a simulated race track. It is shown that our proposed strategy allows the ego vehicle to succeed in overtaking tasks when there are multiple vehicles moving around the ego vehicle.
II. BACKGROUND

In this section, we revisit the vehicle model and learning-based MPC for iterative tasks. The learning-based MPC will be used as the trajectory planner when no surrounding vehicles exist.

A. Vehicle Model

In this work, we use a dynamic bicycle model under Frenet coordinates. The system dynamics is described as follows,

\[ \dot{x} = f(x, u), \]  

where \( x \) and \( u \) show the state and input of the vehicle, \( f \) is a nonlinear dynamic bicycle model in [38]. The definition of state and input is as follows,

\[ x = [v_x, v_y, \omega_z, e_\psi, s_c, e_y]^T, \quad u = [a, \delta]^T, \]  

where acceleration at vehicle’s center of gravity \( a \) and steering angle \( \delta \) are the system’s inputs. \( s_c \) denotes the curvilinear distance travelled along the track’s center line, \( e_y \) and \( e_\psi \) show the deviation distance and heading angle error between vehicle and center line. \( v_x, v_y \) and \( \omega_z \) are the longitudinal velocity, lateral velocity and yaw rate, respectively.

In this paper, this model [1] is applied for precise numerical simulation using Euler discretization with sampling time 0.001s (1000Hz). Through linear regression from the simulated reference path, an affine time-invariant model as below,

\[ x_{t+1} = A(x)x_t + B(x)u_t \]  

will be used in the trajectory planner to avoid excessive complexity from nonlinear optimization, where \( \hat{x} \) represents the equilibrium point for linearized dynamics. On the other hand, an affine time-varying model as below,

\[ x_{t+1} = A_t(\bar{x}_k)x_t + B_t(\bar{x}_k)u_t + C_t(\bar{x}_k) \]

where matrices \( A_t(\bar{x}_k), B_t(\bar{x}_k), \) and \( C_t(\bar{x}_k) \) are obtained at local equilibrium point \( \bar{x}_k \) on reference trajectory with iterative data which is close to \( x_t \). The dynamics [4] will be used on racing controller design for better tracking performance.

The data collection for iterative tasks for reference trajectory will be presented in the next part.

B. Iterative Learning Control

A learning-based MPC [15], which improves the ego vehicle’s lap timing performance through iterative tasks, will be used in this paper. This has the following components:

1) Data Collection: The learning-based MPC optimizes the lap timing through historical states and inputs from iterative tasks. To collect initial data, a simple tracking controller like PID or MPC can be used for the first several laps. During the data collection process, after the \( j \)-th iteration (lap), the controller will store the ego vehicle’s closed-loop states and inputs as vectors. Meanwhile, through offline calculation, every point of this iteration will be associated with a cost, which describes the time to finish the lap from this point.

2) Online Optimization: After the initial laps, the learning-based MPC optimizes the vehicle’s behavior based on collected data. At each time step, a convex set for terminal constraint (green convex hull in Fig. 3) is built to represent the states that can drive the ego vehicle to the finish line in the previous laps. By constructing the cost function to create a minimum-time problem, an open-loop optimized trajectory can be generated. Since the cost function is based on the previous states’ timing data, the vehicle is able to drive to the finish line with time that is no greater than the time from the same position during previous laps. As a result, the ego vehicle will reach the time-optimal performance after several laps.

More details of this method can be found in [15]. In our work, this approach will be used for trajectory planning when the ego vehicle has no surrounding vehicles. This helps with better lap timing without surrounding vehicles. Notice that the data for iterative learning control will be collected through offline simulation with no obstacles on the track, shown in Fig. 2.

III. RACING ALGORITHM

After introducing the background of vehicle modeling and learning-based MPC, we will present an autonomous racing strategy that can help the ego vehicle enhance lap timing performance while overtaking other moving vehicles.
A. Autonomous Racing Strategy

There are two tasks in autonomous racing: enhancing the lap timing performance and competing with other vehicles. To deal these two problems, our proposed strategy will switch between two different planning strategies. When there are no surrounding vehicles, trajectory planning with learning-based MPC is used to enhance the timing performance through historical data. Once the leading vehicles are close enough, an optimization-based trajectory planner optimizes several homotopic trajectories in parallel and the collision-free optimal trajectory is selected with an optimal-time criterion, which will be tracked by a low-level MPC controller. By adding obstacle avoidance constraints to the low-level controller, it has the ability to guarantee the system’s safety. The racing strategy is summarized in Fig. 2.

B. Overtaking Planner

To determine if a leading vehicle is in the ego vehicle’s range of overtaking, the condition must be satisfied:

\[-\ell_l \leq s_{e,c,i} - s_e \leq \ell_l + \gamma |v_x - v_{x,i}|\]  

(5)

where \(s_e\) and \(s_{e,c,i}\) are ego vehicle’s and \(i\)-th surrounding vehicle’s traveling distance, \(v_x\) and \(v_{x,i}\) are ego vehicle’s and \(i\)-th surrounding vehicle’s longitudinal speed. \(l\) indicates the vehicle’s length, \(\epsilon\) and \(\gamma\) are safety-margin factor and prediction factor which we can tune for different performance.

As shown in Fig. 4 when there are \(n\) vehicles in the ego vehicle’s range of overtaking, there exists \((n + 1)\) potential areas, each leading to paths with a different homotopy, that the ego vehicle can use to overtake these surrounding vehicles. These areas are the one below the \(n\)-th vehicle, the one above the 1st vehicle, and the ones between each group of adjacent vehicles. \(n+1\) groups of optimization-based trajectory planning problems are solved in parallel, enabling steady low computational complexity even when competing with different numbers of surrounding vehicles. To reduce each optimization problem’s computational complexity, geometric paths with a distinct homotopy class that laterally lay between vehicles or vehicle and track boundary (black dashed curves in Fig. 4) are used as reference paths in the optimization problems. By comparing the optimization problems’ costs, the optimal trajectory is selected from \(n+1\) optimized solutions. For example, as the case shown in Fig. 4, the dashed orange line in area 2 will be selected since it avoids surrounding vehicles and finishes overtake maneuver with smaller time.

The function to minimize during the selection is shown as follows,

\[
J_s(x_t) = \min_{x_t} -K_s(s_{c,t,N} - s_{c,t}) - \sum_{k=1}^{N_p} ((s_{c,t+k} - s_{c,t+k} - s_{c,t+k+1})^2 - d^2 + b
\]

(6)

where \(K_s\) is a scalar used in metric for timing and \(b\) is a non-zero penalty cost if the new potential area of overtaking is different from the area of overtaking in the last time step. A bigger value of \(K_s\) is applied such that the ego vehicle is optimized to reach a farther point during the overtake maneuver, which results in a shorter overtaking time since the planner’s prediction horizon and sampling time are fixed. Additionally, the other terms in (6) prevents the ego vehicle from changing direction abruptly during an overtake maneuver and guarantees the ego vehicle’s safety.

Bezier-curves are widely used in path planning algorithms in autonomous driving research [39]–[41] because it is easy...
to tune and formulate. Third-order Bezier-curves are used in this work. Each Bezier-curve is interpolated from four control points, including shared start and end points with two additional intermediate points, shown in Fig. 5. Specifically, the start point for the Bezier curve is the ego vehicle’s current position and end point is on the time-optimal trajectory generated from learning-based MPC planner. The selection of end point makes vehicle’s state as close as possible to the time-optimal trajectory after overtake behavior. To make all curves smoother and have no or fewer conflicts with the surrounding vehicles, the other two control points will be between the track’s boundary and vehicle for Areas 1 and n+1 shown in Fig. 5a or between two adjacent vehicles shown in Fig. 5b. These two intermediate control points will have the same lateral deviation from the center line. The key advantage of our selection of control points is that the interpolated geometric curve won’t cross the connected lines between control points with its convexity, shown in Fig. 5b. This property makes our reference paths collision-free with respect to other surrounding vehicles in most cases, which speed up the computational time of the trajectory generation at each area.

**Remark 1.** When the ego vehicle is approaching other surrounding vehicles with big lateral difference (see Fig. 6), the Bezier-curve path used in planner might be not collision-free with other surrounding vehicles. In this case, the corresponding area is not an ideal choice for the overtake maneuver since it asks the ego vehicle to change direction abruptly. Therefore, the optimized trajectory is either infeasible or has a large overtake time and is consequently not selected.

**Remark 2.** In order to ultimately speed up convergence of the optimization problem, the ideal choice for reference paths in the optimization problem should be a collision-free path for the ego vehicle. However, this comes with a high computational complexity with nonlinear optimization. Therefore, we use the curves that has no or fewer conflicts with other vehicles instead of the collision-free curves in this paper to find an appropriate trade-off between complexity and performance of optimized trajectory. Spline curves like cubic curvature polynomials, Dubin’s paths and Bezier curves are suitable candidate types for reference paths.

The details of the optimization formulation for trajectory generation will be illustrated in the next section.

### C. Trajectory Generation

After illustrating the planning strategy, this subsection will show the details about the optimization problem used for trajectory generation for each potential area with different homotopic paths that the ego vehicle can use to overtake the surrounding vehicles.

The optimization problem is formulated as follows,

\[
\begin{align*}
\arg\min_{x_{t+k}, u_{t+k}} & \quad p(x_{t+N}) + \sum_{k=0}^{N_p} q(x_{t+k}) \\
\text{s.t.} & \quad x_{t+k+1|t} = Ax_{t+k|t} + Bu_{t+k|t}, k = 0, ..., N_p-1 \\
& \quad x_{t+k+1|t} \in \mathcal{N}, u_{t+k|t} \in \mathcal{U}, k = 0, ..., N_p-1 \\
& \quad x_t = x_t, \\
& \quad g(x_{t+k}) \geq d + \epsilon, k = 0, ..., N_p-1
\end{align*}
\]

where (7a), (7b), (7d) are constraints for system dynamics, state/input bounds and initial condition. The system dynamics constraint describes the affine linearized model described in (6). The cost function (7a) is composed with three parts, the terminal cost \( p(x_{t+N}) \), the stage cost \( q(x_{t+k}) \) and the state/input changing rate cost \( r(x_{t+k}, u_{t+k}) \). The construction of cost function and constraints in the optimization will be presented in details in the following subsections.

1) **Terminal Cost:** Terminal cost is about the ego vehicle’s traveling distance along the track during the overtaking process

\[
p(x_{t+N}) = K_d(s_{c+t+N} - s_{c_t})
\]

This compares the open-loop predicted traveling distance at the \( N \)-th step \( s_{c+t+N} \) with the ego vehicle’s current traveling distance \( s_{c_t} \). This works as the cost metric for timing during the overtaking process.

2) **Stage Cost:** The stage cost introduces the lateral position differences between the open-loop predicted trajectory and other two paths along the horizon.

\[
q(x_{t+k}) = ||x_{t+k} - x_R(s_{c_t})||_Q + ||x_{t+k} - x_T(s_{c_t})||_Q
\]

\( x_R \) and \( x_T \) are the reference path and time-optimal trajectory in Frenet coordinates. The time-optimal trajectory is generated by the learning-based MPC trajectory planner used on a track without other agents, discussed in Sec. II-B. \( s_{c_t} \) is an initial guess for the traveling distance at the \( k \)-th step, which is equal to \( s_{c_t} = s_{c_t} + v_{c_t} k \Delta t \), where a constant longitudinal speed is assumed along the prediction horizon.

3) **State/Input Changing Rate Cost:** To make the predicted trajectory smoother, the state/input changing rate cost \( r(x_{t+k}, u_{t+k}) \) is formulated as follow:

\[
r(x_{t+k}, u_{t+k}) = ||x_{t+k} - x_{t+k-1}||_H + ||u_{t+k} - u_{t+k-1}||_H
\]
4) Obstacle Avoidance Constraint: In order to generate a collision-free trajectory, collision avoidance constraint \((7c)\) is added in the optimization problem. To reduce computational complexity, only linear lateral position constraint will be added when the ego vehicle overlaps with other vehicles longitudinally. \(|s_{c}(t) + v_{x}(t)Δt - s_{c,i}(t + k)| < l + ε\) will be used to check if the ego vehicle is overlapping with other vehicle longitudinally along the horizon. In \((7c)\), \(g(x) = |e_{y,i} - e_{y}|\) shows the lateral position difference, \(l\) and \(d\) are the vehicle’s length and width, \(ε\) is a safe margin.

After parallel computation, the optimized trajectory \(x_{s,t+\Delta t}^{*}\) with the minimum cost \(J_{s}(x_{1})\) discussed in \((6)\) will be selected from \(n + 1\) groups of optimization problems. This trajectory will be tracked by the MPC controller introduced in \((11)\).

### D. Overtaking Controller

After introducing the algorithm for trajectory generation, a low-level tracking controller with model predictive control used for overtaking will be discussed in this part. The constrained optimization problem is described as follows:

\[
\begin{align*}
\arg\min_{\tilde{u}_{t+1}, t+1} & \quad \sum_{k=0}^{N_{c}-1} q(\tilde{x}_{t+k|t}, \tilde{u}_{t+k|t}) + p_{\omega}(1 - \omega_{k})^{2} \\
\text{s.t.} & \quad \tilde{x}_{t+k+1|t} = A_{t+k|t}\tilde{x}_{t+k|t} + B_{t+k|t}\tilde{u}_{t+k|t} \\
& \quad + C_{t+k|t}, \quad k = 0, ..., N_{c}-1 \\
& \quad \tilde{x}_{t+k|t} \in \tilde{X}, \tilde{u}_{t+k|t} \in \tilde{U}, \quad k = 0, ..., N_{c}-1 \\
& \quad h(\tilde{x}_{t+k+1|t}) \geq \gamma_{w_{k}}h(\tilde{x}_{t+k|t}), \quad k = 0, ..., N_{c}-1
\end{align*}
\]

\((11a)-(11e)\)

where \((11b), (11c), (11d)\) describe the constraints for system dynamics, input state bounds and initial conditions, respectively. The \(q(\tilde{x}_{t+k|t}, \tilde{u}_{t+k|t}) = ||\tilde{x}_{t+k|t} - x_{s,t+k|t}||_{Q_{1}}^{2}\) represents the stage cost, which tracks the desired trajectory \(x_{s,t+\Delta t}^{*}\) optimized by the trajectory planner. Equation \((11e)\) with \(0 \leq \gamma < 1\) represents discrete-time control barrier function constraints \((42)\) with relaxation ratio \(\omega_{k}\), which could guarantee the system’s safety by guaranteeing \(h(\tilde{x}_{t+k|t}) > 0\) along the horizon with forward invariance. In this project, \(h(\tilde{x}_{t+k|t}) = (\bar{s}_{c,i} - \bar{s}_{c})^{2} + (\bar{e}_{y,i} - \bar{e}_{y})^{2} - l^{2} - d^{2}\) is used to represent the distance between the ego vehicle and other vehicles.

The optimization \((11)\) allows us to find the optimal control \(u_{t}^{*} = \tilde{u}_{t|t}\) in a manner similar to MPC.

### IV. RESULTS

#### A. Simulation Setup

Having illustrated our autonomous racing strategy and algorithm, we are going to validate our approach through numerical simulations in this section. In all simulations, all vehicles are with a length of 0.4m and a width of 0.2m. The track’s width is set to 2 m. The horizon lengths for trajectory planner and controller are \(N_{p} = 12, N_{c} = 10\) and shared discretization time \(\Delta t = 0.1s\). Both state and input noises are considered in the simulations. The optimization problems \((7)\) and \((11)\) are implemented in Python with CasADI \([43]\) used as modeling language, are solved with IPOPT \([44]\) on Ubuntu 18.04 on a laptop with a CPU i7-9850 processor at a 2.6GHz clock rate.

#### B. Racing With Other Vehicles

Snapshots shown in Fig. 7 illustrate examples of overtaking behavior in both straight and curvy track segments when competing with other vehicles. When the ego vehicle competes with three surrounding vehicles, it could overtake them on one side of all vehicles (Fig. 7a) or between them (Fig. 7b). The animations of more challenging overtaking behavior can be found on \url{https://youtu.be/41r0V-u6rf4} As shown in TABLE I, the proposed racing
planner could update at 25 Hz and could help the ego vehicle overtake multiple moving vehicles. By switching to a trajectory planning based on learning-based MPC, the ego vehicle is able to reach its speed and steering limit when there are no surrounding vehicles.

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Speed Range [m/s]} & 0 - 0.4 & 0.4 - 0.8 & 0.8 - 1.2 & 1.2 - 1.6 \\
\hline
\text{mean [s]} & 1.613 & 2.312 & 3.857 & 13.095 \\
\text{min [s]} & 0.8 & 1.2 & 1.8 & 3.5 \\
\text{max [s]} & 3.6 & 5.2 & 21.6 & 36.1 \\
\hline
\end{array}
\]

**TABLE II:** Overtaking time for leading vehicle with different speeds. For each group of speed range of the leading vehicles, 100 cases were simulated. The ego vehicle starts from the track’s origin. Other competing vehicle starts from a random position in the range of \(10m \leq s_{e,i} \leq 30m\). The mean, min and max values show the average overtaking time, minimum overtaking time and maximum overtaking time for the corresponding group.

To better analyze the performance and limitations of our autonomous racing strategy in different scenarios, random tests are introduced under two groups. The first group of simulation aims to show the overtaking time for passing one leading vehicle with different speeds, and statistical results are summarized in **TABLE II**. We can observe that when the surrounding vehicles’ speed reaches between 1.2m/s and 1.6m/s, much more time is needed for the ego vehicle to overtake the leading vehicle. This is because as the leading vehicle’s speed increases, less space becomes available for the ego vehicle to drive safely. Especially in a curve, the ego vehicle’s speed limit decreases when less space can be used to make a turn. Since more than half of our track is with curves, the ego vehicle needs to wait for a straight segment to accelerate to pass the leading vehicle.

<table>
<thead>
<tr>
<th>Speed Range [m/s]</th>
<th>0 - 0.4</th>
<th>0.4 - 0.8</th>
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<tr>
<td>Single</td>
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<td>Two</td>
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<tr>
<td>Three</td>
<td>100 %</td>
<td>98 %</td>
<td>84 %</td>
<td>36 %</td>
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</table>

**TABLE III:** Overtaking success rate for the ego vehicle after one lap. For each group of speed range of the leading vehicles, 100 cases were simulated. The ego vehicle starts from the track’s origin. Other vehicles start from a random position in the range of \(5m \leq s_{e,i} \leq 15m\). One to three leading cars were simulated.

The second group of simulation shows the proposed racing strategy’s success rate to overtake multiple leading vehicles in one lap, and statistical results are summarized in **TABLE III**. We can find that when more than one surrounding vehicle exists, much more space would be occupied by other vehicles. As a result, the ego vehicle might not have enough space to accelerate to high speed to pass surrounding vehicles. Although in these cases, the ego vehicle can not overtake all surrounding vehicles after one lap, our proposed racing strategy can still guarantee the ego vehicle’s safety along the track.

During our simulation, the mean solver time for our planner for single, two or three surrounding vehicles is 39.21ms, 39.41ms and 40.23ms. We also notice that when the number of surrounding vehicles is larger than three, the steady complexity still holds but the track becomes too crowded for the ego vehicle to achieve high success rate of overtake maneuver. This validates the steady low computational complexity of proposed planning strategy thanks to the parallel computation for multiple trajectory optimizations.

To verify the performance of our proposed algorithm through numerical simulation, where several surrounding vehicles are simulated to start from random position with random speed and steering limit, we can observe that when more than one surrounding vehicles are simulated to start from random position with random speed and steering limit, the ego vehicle could update at 25 Hz and could help the ego vehicle overtake multiple moving vehicles. By switching to a trajectory planning based on learning-based MPC, the ego vehicle is able to reach its speed and steering limit when there are no surrounding vehicles. We also notice that when the number of surrounding vehicles is larger than three, the steady complexity still holds but the track becomes too crowded for the ego vehicle to achieve high success rate of overtake maneuver. This validates the steady low computational complexity of proposed planning strategy thanks to the parallel computation for multiple trajectory optimizations.

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