Controlling the Locomotion of Spherical Robots or why BB-8 Works

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Spherical robots have a wide range of self-propulsion mechanisms. Of particular interest in this paper, are propulsion systems where wheels are placed in contact with the inner surface of the spherical shell of the robot. Here, locomotion is achieved by a combination of the actions of the motors along with the rolling constraints at the point of contact of the shell with the ground surface. We ask and seek the answer to the following question using elementary arguments: What is the minimal number of actuations needed to completely prescribe the motion of the robot for the two distinct cases where it is rolling and sliding on a surface? We find that two points of actuation are all that is needed provided some simple geometric conditions are satisfied. Our analysis is then applied to the BB-8 robot to show how locomotion is achieved in this robot.

1 Introduction

Mechanical toys featuring spherical shells that are propelled into motion on surfaces using a wide range of internal mechanisms date to the early 1900s [1]. These toys partially inspired the development of spherical robots later in the same century, developments which continue to the present day. This class of robots have a spherical shell and a propulsion mechanism contained within the shell (see, e.g., [2, 3, 4, 5]). Locomotion is achieved by the rolling of the shell on a surface. The analyses of the dynamics of the rolling motion and controlling and planning the trajectory has a rich history (cf. [6, 7, 8, 9, 10] and references therein) and has raised many interesting questions in the dynamics and control of nonholonomic systems. Our present work contributes to this literature by seeking simple explanations behind the kinematics needed to control a spherical robot.

It can be argued that spherical robots were largely of academic interest until the introduction of the *Sphero-based BB-8 robot* in the *Star Wars: The Force Awakens* movie and its sequel captured the public imagination. From the patents [11, 12, 13] for this robot and disassembling a working device (see Figure 1), we learn that self-propulsion is achieved in a similar manner to how a hamster is able to move its cage albeit using two wheels in contact with the inner surface of the shell.[1] The tilting of the external spherical cap (domed head), which is arguably its defining feature, is achieved using magnets and a pendulum-like actuator.

Our curiosity about the self-propulsion system used by BB-8 leads us to ask the following questions: First, how many actuators are needed to control the motion of such a device? Second, is it necessary for BB-8 to roll or is it also possible to control the motion if the robot is sliding? While we didn't find the answers to our questions in the literature, we were able to find answers using elementary arguments. In short, we find that it is necessary to prescribe the motion of two distinct points on a rigid body and the motion of a third point in a single direction in order to completely control the motion. Thus, we conclude that two actuators (as opposed to four in the design of some spherical robots) are required to locomote a robot, even when the robot is sliding. We conclude the paper with an application of our results to BB-8.

[1] A video [14] showing the inner workings of the Sphero BB-8 robot proved to be very helpful.

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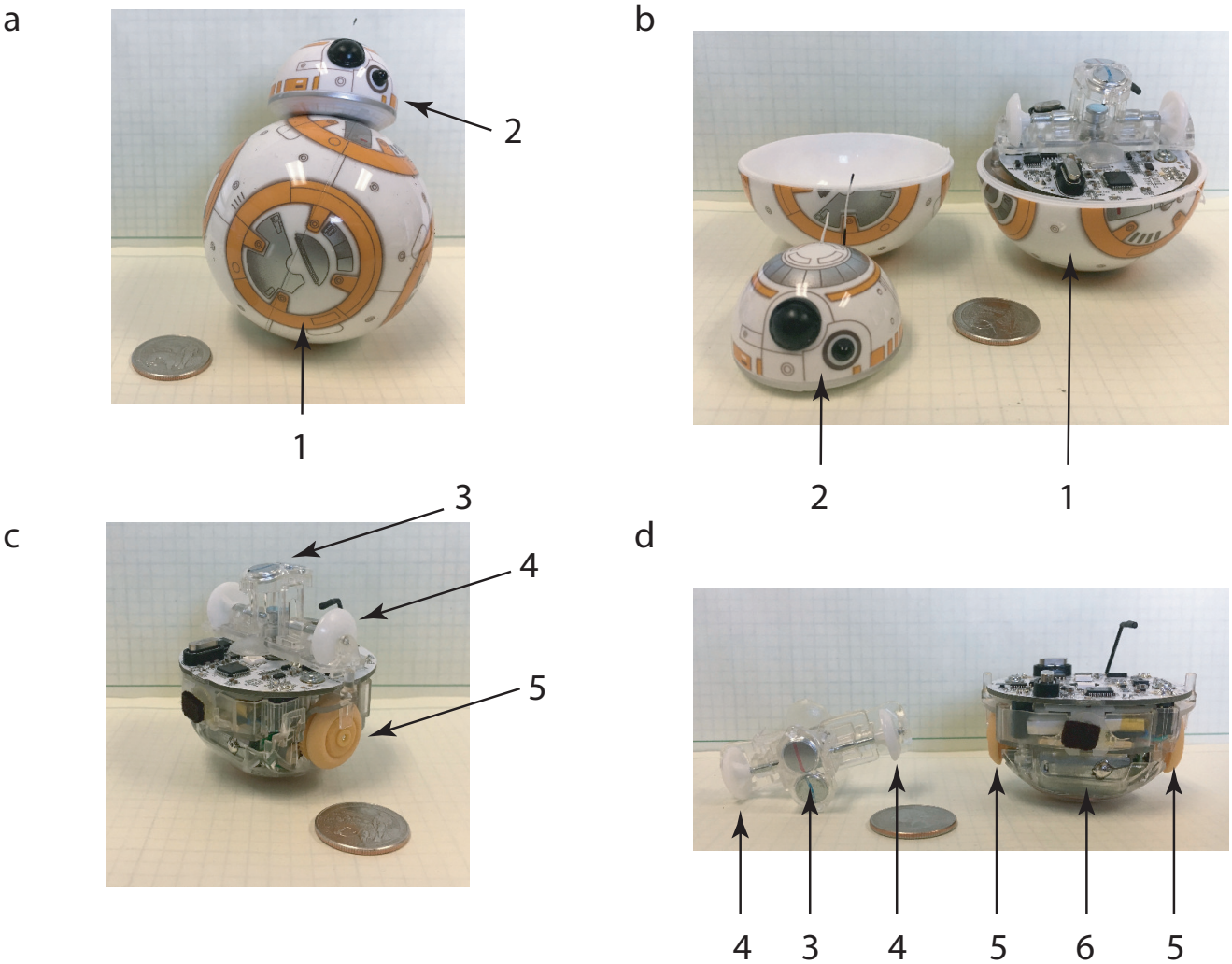

Fig. 1: The Sphero BB-8 robot. (a) The robot in motion showing the spherical shell (1) and the spherical cap. (b) Deconstructed robot following cutting of the spherical shell into hemispheres and removal of cap (2). (c) the inner mechanism of the robot showing one of the two wheels (5) used to locomote the spherical shell and the pair of wheels (4) used to help maintain contact. The pendular mechanism (3) which actuates the spherical cap with the help of magnets is labelled. (d) Image of the inner mechanism showing the pair of wheels used to locomote the spherical shell and the pendular mechanism. By way of scale, the coin in the images is a US quarter dollar. The induction coil (6) used to charge the batteries for the rotors is located at the bottom of the inner mechanism and serves to help keep the center of mass of the robot below the geometric center of the spherical shell

2 Elementary Kinematics

The motion of a rigid body can be described using the position vector $\bar{\mathbf{x}}$ of the center of mass $\bar{X}$ and the rotation tensor $\mathbf{Q}$ of the body. For our purposes it is convenient to frame our discussion in terms of the velocity vector $\bar{\mathbf{v}} = \dot{\bar{\mathbf{x}}}$ of the center of mass and the angular velocity vector $\boldsymbol{\omega}$ of the rigid body. Our notation follows [15].

For an arbitrary material point A of the rigid body, we have the following relationship:

$$\mathbf{v}_A = \bar{\mathbf{v}} + \boldsymbol{\omega} \times (\mathbf{x}_A - \bar{\mathbf{x}}) \,. \tag{1}$$

In the sequel, we use a right-handed orthonormal corotational (or body fixed) basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ and the following representations:

$$\begin{aligned} \bar{\mathbf{v}} &= \dot{x}_1 \mathbf{e}_1 + \dot{x}_2 \mathbf{e}_2 + \dot{x}_3 \mathbf{e}_3 \,, \\ \boldsymbol{\omega} &= \omega_1 \mathbf{e}_1 + \omega_2 \mathbf{e}_2 + \omega_3 \mathbf{e}_3 \,, \\ \mathbf{x}_A - \bar{\mathbf{x}} &= A_1 \mathbf{e}_1 + A_2 \mathbf{e}_2 + A_3 \mathbf{e}_3 \,. \end{aligned} \tag{2}$$

Using a skew-symmetric matrix we can express the velocity equation (1) using arrays and a matrix:

$$\mathsf{v}_A = \bar{\mathsf{v}} - \mathsf{A}\boldsymbol{\omega} \tag{3}$$

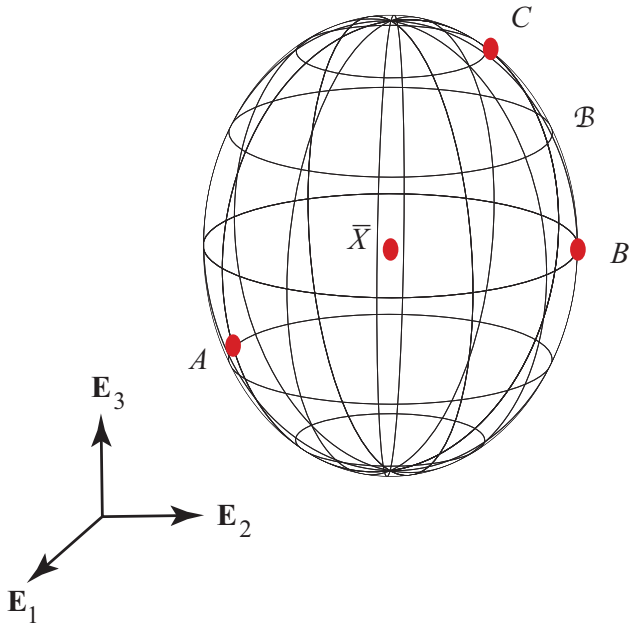

Fig. 2: Schematic of the material points A, B, and C on a rigid body and the center of mass $\bar{X}$ of the rigid body $\mathcal{B}$.The basis $\{\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3\}$ is a fixed right-handed Cartesian basis

where

$$\mathsf{v}_A = \begin{bmatrix} \mathbf{v}_A \cdot \mathbf{e}_1 \\ \mathbf{v}_A \cdot \mathbf{e}_2 \\ \mathbf{v}_A \cdot \mathbf{e}_3 \end{bmatrix}, \qquad \bar{\mathsf{v}} = \begin{bmatrix} \bar{\mathbf{v}} \cdot \mathbf{e}_1 \\ \bar{\mathbf{v}} \cdot \mathbf{e}_2 \\ \bar{\mathbf{v}} \cdot \mathbf{e}_3 \end{bmatrix},$$
$$\mathsf{A} = \begin{bmatrix} 0 & -A_3 & A_2 \\ A_3 & 0 & -A_1 \\ -A_2 & A_1 & 0 \end{bmatrix}, \qquad \boldsymbol{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}. \tag{4}$$

The san-serif font is used to distinguish scalar valued arrays and matrices from the vector and tensor counterparts.

Given three points A, B, and C on a rigid body, such as those shown in Fig. 2, we have the following identities:

$$\begin{aligned} \mathbf{v}_A - \mathbf{v}_C &= \boldsymbol{\omega} \times (\mathbf{x}_A - \mathbf{x}_C), \\ \mathbf{v}_B - \mathbf{v}_C &= \boldsymbol{\omega} \times (\mathbf{x}_B - \mathbf{x}_C). \end{aligned} \tag{5}$$

The relative velocities in these expressions are not independent. After taking an inner product and rearranging the resulting triple product, we find that

$$(\mathbf{v}_A - \mathbf{v}_C) \cdot (\mathbf{x}_B - \mathbf{x}_C) = -(\mathbf{v}_B - \mathbf{v}_C) \cdot (\mathbf{x}_A - \mathbf{x}_C). \tag{6}$$

Thus, at most five of the six scalar relations (5) are independent. We also note for future reference that the relative velocity vectors and relative position vectors are orthogonal:

$$\begin{aligned} (\mathbf{v}_A - \mathbf{v}_C) \cdot (\mathbf{x}_A - \mathbf{x}_C) &= 0, \\ (\mathbf{v}_B - \mathbf{v}_C) \cdot (\mathbf{x}_B - \mathbf{x}_C) &= 0. \end{aligned} \tag{7}$$

3 Prescribing the Motions of Two Points

Imagine holding a rigid body in space with two fingers and then attempting to move the body using these fingers. It is easy to see that while some of the motion of the body can be controlled in this manner, it is not possible to control a motion where the body rotates about an axis that joins the two fingers. Similarly, if BB-8 were thrown in the air and then the pair of motors were actuated, it would not be possible to control all six components of $\bar{\mathbf{v}}$ and $\boldsymbol{\omega}$.

To see how these limitations manifest in the kinematics of the rigid body, we suppose that the velocity vectors at two points A and B of the rigid body are prescribed:

$$\mathbf{v}_A = \bar{\mathbf{v}} + \boldsymbol{\omega} \times (\mathbf{x}_A - \bar{\mathbf{x}}), \qquad \mathbf{v}_B = \bar{\mathbf{v}} + \boldsymbol{\omega} \times (\mathbf{x}_B - \bar{\mathbf{x}}). \tag{8}$$

We can express these relations as a matrix equation:

$$\begin{bmatrix} \mathsf{v}_A \\ \mathsf{v}_B \end{bmatrix} = \begin{bmatrix} \mathbf{I} & -\mathsf{A} \\ \mathbf{I} & -\mathsf{B} \end{bmatrix} \begin{bmatrix} \bar{\mathsf{v}} \\ \boldsymbol{\omega} \end{bmatrix}. \tag{9}$$

If $(\mathbf{x}_A - \bar{\mathbf{x}}) \times (\mathbf{x}_B - \bar{\mathbf{x}}) \neq \mathbf{0}$, then the square matrix in Eqn. (9) has rank 5 and a non-zero null vector. The null vector corresponds to

$$\bar{\mathbf{v}} = \lambda (\mathbf{x}_A - \bar{\mathbf{x}}) \times (\mathbf{x}_B - \bar{\mathbf{x}}), \qquad \boldsymbol{\omega} = \lambda (\mathbf{x}_B - \mathbf{x}_A), \tag{10}$$

where λ is a scalar. Thus, given $\mathbf{v}_A$ and $\mathbf{v}_B$, we can prescribe all of the components of $\bar{\mathbf{v}}$ and $\boldsymbol{\omega}$ except those for a motion where the body rotates at an arbitrary angular speed about an axis passing through the fixed points A and B (i.e., $\mathbf{v}_A = \mathbf{0}$, $\mathbf{v}_B = \mathbf{0}$, and $\bar{\mathbf{v}} \perp \mathbf{x}_A - \mathbf{x}_B$).

4 Prescribing the Motions of Three Points

To attempt to eliminate the uncontrolled motion that appears when the motions of two points are prescribed, we next suppose that the velocity vector at three points A, B, and C of the rigid body are prescribed:

$$\begin{aligned} \mathbf{v}_A = \bar{\mathbf{v}} + \boldsymbol{\omega} \times (\mathbf{x}_A - \bar{\mathbf{x}}), \qquad \mathbf{v}_B = \bar{\mathbf{v}} + \boldsymbol{\omega} \times (\mathbf{x}_B - \bar{\mathbf{x}}), \\ \mathbf{v}_C = \bar{\mathbf{v}} + \boldsymbol{\omega} \times (\mathbf{x}_C - \bar{\mathbf{x}}). \end{aligned} \tag{11}$$

Writing these equations in a matrix notation:

$$\begin{bmatrix} \mathsf{v}_A \\ \mathsf{v}_B \\ \mathsf{v}_C \end{bmatrix} = \begin{bmatrix} \mathbf{I} & -\mathsf{A} \\ \mathbf{I} & -\mathsf{B} \\ \mathbf{I} & -\mathsf{C} \end{bmatrix} \begin{bmatrix} \bar{\mathsf{v}} \\ \boldsymbol{\omega} \end{bmatrix}. \tag{12}$$

The 9×6 matrix in this equation has rank 6. It row reduces to a matrix which has a block structure consisting of a 6×6 identity matrix and a 3×6 zero matrix.

Without loss of generality, we can assume that the velocities we wish to control are

$$\boldsymbol{\omega} \text{ and } \mathbf{v}_C. \tag{13}$$

That is, we only need to determine $\boldsymbol{\omega}$. Suppose we now prescribe $\mathbf{v}_B$ and $\mathbf{v}_A$. Then,

$$\mathbf{v}_A - \mathbf{v}_C = \boldsymbol{\omega} \times (\mathbf{x}_A - \mathbf{x}_C), \qquad \mathbf{v}_B - \mathbf{v}_C = \boldsymbol{\omega} \times (\mathbf{x}_B - \mathbf{x}_C). \tag{14}$$

As mentioned earlier, the relative velocities in Eqn. (14) are not independent (cf. Eqn. (6)). Without loss of generality, we choose $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ such that

$$\mathbf{x}_A - \mathbf{x}_C = D_3 \mathbf{e}_3. \tag{15}$$

We still need to compute $\boldsymbol{\omega}$ in order to determine the motion of the body using the remaining equations:

$$\begin{bmatrix} \mathsf{v}_A - \mathsf{v}_C \\ \mathsf{v}_B - \mathsf{v}_C \end{bmatrix} = \begin{bmatrix} -\mathsf{A}_1 \\ -\mathsf{B}_1 \end{bmatrix} \begin{bmatrix} \omega \end{bmatrix}, \tag{16}$$

where

$$\mathsf{A}_1 = \begin{bmatrix} 0 & -D_3 & 0 \\ D_3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\mathsf{B}_1 = \begin{bmatrix} 0 & -B_3 + C_3 & B_2 - C_2 \\ B_3 - C_3 & 0 & -B_1 + C_1 \\ -B_2 + C_2 & B_1 - C_1 & 0 \end{bmatrix}. \tag{17}$$

The null spaces of A_1 and B_1 are column vectors composed of the components of $\mathbf{x}_A - \mathbf{x}_C$ and $\mathbf{x}_B - \mathbf{x}_C$, respectively. Provided A, B, and C are not co-linear, that is

$$(\mathbf{x}_A - \mathbf{x}_C) \times (\mathbf{x}_B - \mathbf{x}_C) \neq \mathbf{0}, \tag{18}$$

then the 6×3 matrix in (16) has rank 3. We can solve for ω_1 and ω_2 using the equation $\mathbf{v}_A - \mathbf{v}_C = \boldsymbol{\omega} \times (\mathbf{x}_A - \mathbf{x}_C)$:

$$\omega_1 = \hat{\omega}_1 \equiv -\frac{(\mathbf{v}_A - \mathbf{v}_C) \cdot \mathbf{e}_2}{D_3}, \qquad \omega_2 = \hat{\omega}_2 \equiv \frac{(\mathbf{v}_A - \mathbf{v}_C) \cdot \mathbf{e}_1}{D_3}. \tag{19}$$

The solutions are ornamented with a hat for future reference. The remaining three scalar equations from Eqn. (16) now reduce to

$$\begin{bmatrix} (\mathbf{v}_B - \mathbf{v}_C) \cdot \mathbf{e}_1 \\ (\mathbf{v}_B - \mathbf{v}_C) \cdot \mathbf{e}_2 \\ (\mathbf{v}_B - \mathbf{v}_C) \cdot \mathbf{e}_3 \end{bmatrix} = -\mathsf{B}_1 \begin{bmatrix} \frac{-(\mathbf{v}_A - \mathbf{v}_C) \cdot \mathbf{e}_2}{D_3} \\ \frac{(\mathbf{v}_A - \mathbf{v}_C) \cdot \mathbf{e}_1}{D_3} \\ \omega_3 \end{bmatrix}. \tag{20}$$

Solving for ω_3, we find

$$\omega_3 = \hat{\omega}_3 \equiv \frac{1}{(B_2 - C_2)} \left[(B_3 - C_3)\, \hat{\omega}_2 - (\mathbf{v}_B - \mathbf{v}_C) \cdot \mathbf{e}_1 \right]. \tag{21}$$

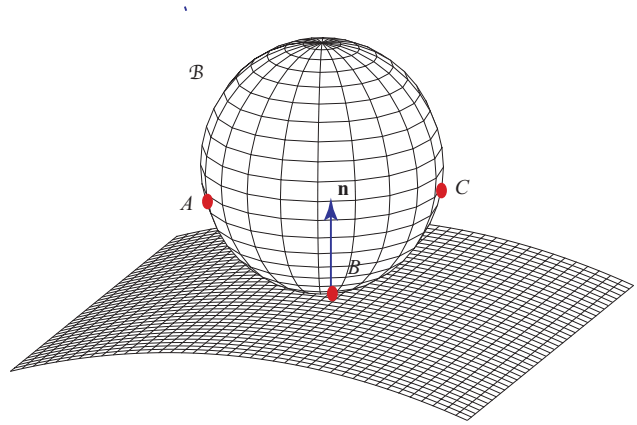

Fig. 3: A rigid body in motion on a surface. The instantaneous point of contact of the body with the surface is B and the normal vector at the contact point is $\mathbf{n}$

Thus, by prescribing the velocity vectors of three non-colinear points, the motion of the rigid body is completely prescribed.

Observe that we have two remaining equations from Eqn. (16):

$$(\mathbf{v}_B - \mathbf{v}_C) \cdot \mathbf{e}_2 = -(B_3 - C_3)\, \hat{\omega}_1 + (B_1 - C_1)\, \hat{\omega}_3,$$
$$(\mathbf{v}_B - \mathbf{v}_C) \cdot \mathbf{e}_3 = (B_2 - C_2)\, \hat{\omega}_1 - (B_1 - C_1)\, \hat{\omega}_2. \tag{22}$$

As anticipated from the rank calculation for the 6×3 matrix in Eqn. (16), the pair of equations (22) is identically satisfied by the solutions $\hat{\omega}_i$ for ω_i. To see this, we rearrange Eqn. (22) and use the identities (7) to find that the two equations (22) can be expressed in the respective forms:

$$(B_3 - C_3)\left((\mathbf{v}_A - \mathbf{v}_C) \cdot (\mathbf{x}_B - \mathbf{x}_C) + (\mathbf{v}_B - \mathbf{v}_C) \cdot (\mathbf{x}_A - \mathbf{x}_C)\right) = 0,$$
$$(\mathbf{v}_A - \mathbf{v}_C) \cdot (\mathbf{x}_B - \mathbf{x}_C) + (\mathbf{v}_B - \mathbf{v}_C) \cdot (\mathbf{x}_A - \mathbf{x}_C) = 0. \tag{23}$$

Invoking Eqn. (6), we can conclude that Eqn. (22) are identically satisfied.

5 The Minimal Case

The fact that two of the nine conditions arising when the velocity vectors of three points are prescribed are redundant naturally leads us to seek the minimal set of prescriptions. To this end, we now consider the case where the velocity vectors of two points A and C are prescribed but only one component of B is prescribed. This situation arises when a spherical robot is in sliding contact with a surface. In this case, we prescribe

$$\mathbf{v}_A, \ \mathbf{v}_C, \text{ and } \mathbf{v}_B \cdot \mathbf{n}, \tag{24}$$

where $\mathbf{n}$ is the unit-normal vector to the surface at the instantaneous point of contact B of the robot with the surface (cf. Fig. 3).

Without loss in generality, we can assume that the velocities we wish to control are $\boldsymbol{\omega}$ and $\mathbf{v}_C$. Paralleling the arguments of the previous section, we find that prescribing $\mathbf{v}_A$ and $\mathbf{v}_C$ enables us to compute the components of $\boldsymbol{\omega}$ orthogonal to $(\mathbf{x}_A - \mathbf{x}_C)$ (cf. Eqn. (19)). We still need to prescribe the component of $\boldsymbol{\omega}$ that is parallel to $(\mathbf{x}_A - \mathbf{x}_C)$. We have a single equation remaining that can be used to solve for this component:

$$(\mathbf{v}_B - \mathbf{v}_C) \cdot \mathbf{n} = (\boldsymbol{\omega} \times (\mathbf{x}_B - \mathbf{x}_C)) \cdot \mathbf{n}. \tag{25}$$

We again choose $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ so that Eqn. (15) holds and define a vector $\mathbf{q}$

$$\mathbf{q} = q_1\mathbf{e}_1 + q_2\mathbf{e}_2 + q_3\mathbf{e}_3 = (\mathbf{x}_B - \mathbf{x}_C) \times \mathbf{n}. \tag{26}$$

Whence we find that the remaining unknown component of $\boldsymbol{\omega}$ is

$$\omega_3 = \hat{\omega}_3 = \frac{1}{q_3}\left((\mathbf{v}_B - \mathbf{v}_C) \cdot \mathbf{n} - \hat{\omega}_1 q_1 - \hat{\omega}_2 q_2\right). \tag{27}$$

For the solution to this equation to be defined we require $q_3 \neq 0$. This is equivalent to the condition

$$((\mathbf{x}_A - \mathbf{x}_C) \times (\mathbf{x}_B - \mathbf{x}_C)) \cdot \mathbf{n} \neq 0. \tag{28}$$

In other words, the relative position vectors $\mathbf{x}_A - \mathbf{x}_C$ and $\mathbf{x}_B - \mathbf{x}_C$ and the normal vector $\mathbf{n}$ are linearly independent. When Eqn. (28) is satisfied, the prescription of $\mathbf{v}_C$, $\mathbf{v}_B \cdot \mathbf{n}$, and the two components of $\mathbf{v}_A$ that are orthogonal to $(\mathbf{x}_A - \mathbf{x}_C)$ is necessary and sufficient to prescribe $\bar{\mathbf{v}}$ and $\boldsymbol{\omega} = \sum_{i=1}^{3} \hat{\omega}_i \mathbf{e}_i$ of the rigid body.

6 Application to a Rolling Spherical Robot and a Sliding Spherical Robot

For BB-8 rolling on a surface, actuation of the robot is achieved by two wheels in contact with the inner surface of the spherical shell (see the wheels labeled "5" in Figure 1). Thus only certain components of the velocity vectors of A, B, and C are prescribed. As shown in Figure 4, we label the point of contact of the wheels with the shell by A and C and the instantaneous point of contact as B. Thus $\mathbf{v}_B$ is completely prescribed (i.e., $\mathbf{v}_B = \mathbf{0}$ when the spherical shell is rolling). Without loss of generality, in addition to our earlier selection of $\mathbf{e}_3$, we also choose $\mathbf{e}_1$ so that

$$\mathbf{v}_A - \mathbf{v}_C = v\mathbf{e}_1, \qquad \mathbf{x}_A - \mathbf{x}_C = D_3\mathbf{e}_3. \tag{29}$$

That is $\mathbf{e}_3$ is parallel to the axles of the wheels which contact the spherical shell at A and C. A cursory examination of a BB-8 robot shows that (18) holds. Whence from (19), we can conclude that

$$\boldsymbol{\omega} = \hat{\omega}_2\mathbf{e}_2 + \hat{\omega}_3\mathbf{e}_3 \tag{30}$$

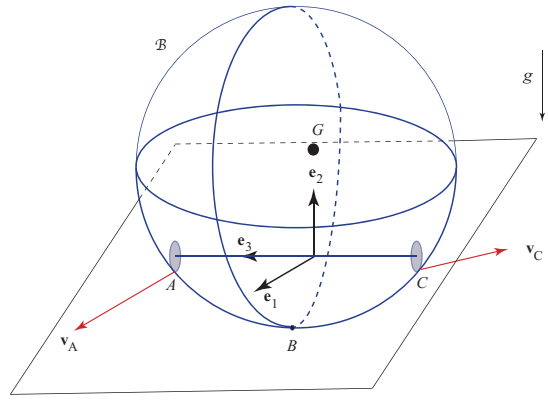

Fig. 4: Schematic of BB-8 in motion on a horizontal plane. The points of actuation of the motor-driven wheels are labelled A and C, the instantaneous point of contact of the shell with the ground plane is labeled B, and the geometric center of the shell is labeled G. The relative velocity vector $\mathbf{v}_A - \mathbf{v}_C = v\mathbf{e}_1$

is prescribed with $\hat{\omega}_1 \equiv 0$.

Given the weight of the structure containing the motors for the wheels and the induction coil, we assume that $\mathbf{e}_2$ is approximately parallel to the position vector of the geometric center G of the spherical shell of radius r_0 relative to the point of contact B: $\mathbf{x}_G - \mathbf{x}_B \approx r_0\mathbf{e}_2$. Whence,

$$\begin{aligned}\mathbf{v}_G &= \mathbf{v}_B + \boldsymbol{\omega} \times (\mathbf{x}_G - \mathbf{x}_B) = -r_0\hat{\omega}_3\mathbf{e}_1 \\ &= \left(\left(\frac{B_3 - C_3}{D_3}\right)(\mathbf{v}_A - \mathbf{v}_C) \cdot \mathbf{e}_1 + \mathbf{v}_C \cdot \mathbf{e}_1\right)\mathbf{e}_1\end{aligned} \tag{31}$$

where we used Eqn. (21) to simplify the resulting expression for $\mathbf{v}_G$. From the expression for $\mathbf{v}_G$ we can conclude that the geometric center will move in a direction orthogonal to the axis connecting A and C. By varying the relative speed of rotation of the wheels at A and C we can rotate $\mathbf{e}_3$ and so control the direction of the motion of G. In this simple manner, the pair of wheels control the trajectory of G.

For the BB-8 robot, an examination of its geometry shows that we do not expect the kinematical condition (28) to hold. Consequently, locomotion of BB-8 on a surface with sliding contact (as opposed to rolling contact) is not anticipated.

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