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DETERMINING MINIMUM AND MAXIMUM NUMBER OF AGENTS REQUIRED FOR PLANAR CABLE-SUSPENDED AERIAL MANIPULATION

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ABSTRACT

This paper proposes a method for optimal selection of number of agents in a cooperative multi-agent system for dynamic aerial transportation of a cable-suspended load along a desired trajectory. We consider the system of n planar quadrotors with a cable suspended load, show that this system is differentially flat, and study the effect of varying the number of quadrotors, load and quadrotor masses, on the maximum velocity of the load trajectory and minimum thrust required. We determine the minimum (maximum) number of agents required for traversing a load trajectory for various values of the load mass.

1 INTRODUCTION

Unmanned Aerial vehicles (UAVs) have been deployed in scenarios such as aerial transportation, search and rescue missions, surveillance, construction, sensing, mapping, etc. In disaster situations and in potentially hazardous regions, where human accessibility is challenging, aerial transportation provides a safe and attractive alternative. Aerial transportation tasks can be achieved using either grippers for grasping [5] or through loads suspended by cables from a single UAV [6, 9, 10], or multiple UAVs [3]. In these cases, as the size of the load increases, a choice must be made on whether to use a single large UAV or several smaller UAVs.

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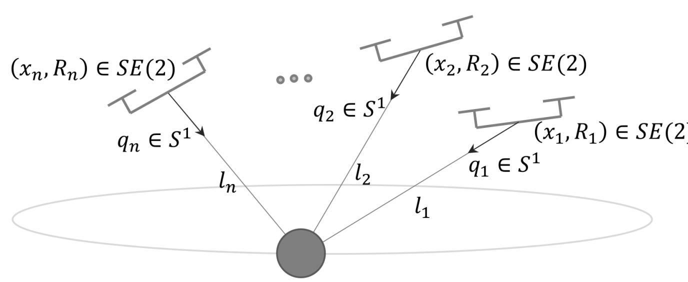

FIGURE 1. Cooperative aerial transportation of a cable-suspended load along a trajectory. This is a multi-agent collaborative task with n agents. An open question in multi-agent systems is: What are the minimum (maximum) number of agents required to perform the multi-agent collaborative task? In this paper, we propose an answer to this question for the task of cooperative aerial transportation.

Dynamic aerial load transportation using multiple cooperative UAVs is a challenging problem. An important and open question is: *What are the minimum (maximum) number of agents required to perform this multi-agent collaborative task?* This question is compounded by the fact that there are several system parameters that can be varied, that include the speed of load trajectory, the mass of the load, the mass of the UAVs, the length of the cables, the maximum lift force of the UAVs, etc. Based on the choice of these variables, it's possible that a single large UAV is better than several small UAVs, or five small UAVs are optimal and better than ten small UAVs.

While multi-agent systems have been used in transportation tasks [4, 11], using planning techniques [12], subject to non-holonomic constraints [13], and communication constraints [2], there is not much work on optimal selection of number of agents for a dynamic cooperative task. However, because of the dynamic setting of our cooperative aerial manipulation task, answering the question of optimal number of agents becomes feasible. In particular, in the task of aerial transportation, the variability in type and size of loads, aerial agents, their thrust capabilities, and associated dynamic trajectories for the load, requires us to systematically study this problem.

In this paper we consider a system of n planar quadrotors with a cable suspended point mass load, and our goal is to determine the minimum number of quadrotors required to transport a specified load through a given dynamic trajectory for the load. This paper investigates the feasibility of a strategy in determining the minimum (maximum) number of agents for multi-agent collaborative systems. It makes strict assumptions about the nature of interaction between the agents so as to verify the applicability of the proposed strategy. We show that this system of quadrotors with a cable suspended load is differentially flat [7, 14], allowing us to express the state and inputs of the system through a set of flat outputs and their higher-order derivatives. Under the assumptions that (a) the angle of separation between the cables is constant, (b) cables are taut and massless, and (c) load trajectories can be expressed as a combination of arcs of ellipses, we can analytically derive the thrust generated by each quadrotor in terms of the quadrotor mass, load mass, cable length, and the desired trajectory. By setting an upper bound on the thrust per quadrotor, we can determine the minimum (maximum) number of quadrotors required to traverse a desired trajectory for the load.

The primary contribution of our paper is to provide a methodology to compute the minimum and maximum number of agents required to perform a cooperative aerial transportation task. We find that the addition of agents past an optimal threshold produces diminishing returns; increasing agent mass has a larger effect on the system than increasing load mass; and that the length of the cable has a non-trivial effect on the system. However, due to the assumptions made, we are unable to address asymmetric problem settings where mass of each quadrotor, length of each cable, separation between each cable could all be different for each agent. Furthermore, we haven't studied varying the maximum thrust of each quadrotor.

The rest of this paper is structured as follows. Section 2 presents the dynamical equations of motion, establishes that our cooperative aerial transportation system is differentially flat, and presents a methodology to determine the minimum (maximum) number of agents required. Section 3 presents results of our computations to study the effect of number of quadrotors, separation angle, load and quadrotor masses on maximum velocity of the load trajectory and minimum thrust required, as well as the effect of cable length on load trajectory and quadrotor thrust. Section 4 presents a detailed discussion on our results and determines the minimum (maximum) number of agents required for a particular load trajectory and as the load mass varies. Finally, Section 5 provides some concluding remarks.

2 METHODOLOGY

In this section, we will present the equations of motion for the planar aerial manipulation problem considered in Figure 1. This system will then be described as a differentially flat system. We will then make certain simplifications that enable an easy choice of the flat outputs, resulting in the load executing an ellipsoidal trajectory in Cartesian space with constant angular velocity. Finally we will analyze how the minimum and maximum number of quadrotors required for achieving this dynamic ellipsoidal trajectory varies as a function of various system parameters, such as load mass, quadrotor mass, length of cables, etc.

2.1 Dynamical Equations of Motion

The planar cable-suspended aerial manipulation system has $2n+2$ degrees of freedom and $2n$ actuators where n is the number of quadrotors. In particular, we have 2 degrees of freedom for the load position, n degrees of freedom corresponding to the cable orientations, and n degrees of freedom corresponding to the quadrotor orientations. Each quadrotor has two actuators (thrust and moment) accounting for the $2n$ actuators in the system. The cables are assumed to be taut and massless. The equations of motion for the full 3D cable-suspended aerial manipulation system are derived using Newton-Euler in [8] and using Lagrange's method in [3]. The Newton-Euler equations of motion can be specialized to the planar case to obtain Equations 1 and 2. Figure 2 gives us the free-body diagram from which Equation 3 follows. For a full derivation please see [8].

$$m_i\ddot{x}_i = f_i R_i e_2 - m_i g e_2 + T_i q_i, \tag{1}$$

$$J_i\ddot{\phi}_i = \tau_i \tag{2}$$

$$m_L\ddot{x}_L = -\Sigma_{i=1}^{n} T_i q_i - m_L g e_2, \tag{3}$$

where the various symbols used here are tabulated in Table 1. In particular, q_i is a unit vector from the i^{th} quadrotor to the load and R_i is the rotation matrix from i^{th} quadrotor's body-fixed frame to the inertial frame, and are as defined below:

$$q_i = \begin{bmatrix} sin(\theta_i) \\ -cos(\theta_i) \end{bmatrix}, \tag{4}$$

TABLE 1. Various symbols being used.

$m_L \in \mathbb{R}$	Mass of the load
$x_L \in \mathbb{R}^2$	Position of the load
$l_i \in \mathbb{R}$	Length of the i^{th} cable
$l \in \mathbb{R}$	When cables are equal length, $l_i = l$
$T_i \in \mathbb{R}$	Tension in the i^{th} cable
$\theta_i \in \mathbb{R}$	Angle of the i^{th} cable wrt vertical
$m_i \in \mathbb{R}$	Mass of the i^{th} quadrotor
$m_Q \in \mathbb{R}$	When quadrotors are equal mass, $m_i = m_Q$
$J_i \in \mathbb{R}$	Inertia of the i^{th} quadrotor
$x_i \in \mathbb{R}^2$	Position of the i^{th} quadrotor
$R_i \in SO(2)$	Orientation of the i^{th} quadrotor
$\phi_i \in \mathbb{R}$	Angle of the i^{th} quadrotor wrt horizontal
$f_i \in \mathbb{R}$	Thrust of the i^{th} quadrotor
$\tau_i \in \mathbb{R}$	Torque of the i^{th} quadrotor
$n \in \mathbb{Z}$	Number of quadrotors
$2\psi \in \mathbb{R}$	Separation angle between cables
$g \in \mathbb{R}$	Acceleration due to gravity
$e_2 \in \mathbb{R}^2$	Unit vector along the y axis

$$R_i = \begin{bmatrix} cos(\phi_i) & -sin(\phi_i) \\ sin(\phi_i) & cos(\phi_i) \end{bmatrix}. \tag{5}$$

Note that θ_i is the angle of the i^{th} cable with respect to the vertical, while ϕ_i is the angle of the i^{th} quadrotor with respect to the horizontal, as shown in Figure 2.

Also note that in the above equations of motion, we have considered certain variables that are dependent on each other. For instance, the position of the i^{th} quadrotor is related to the position of the load through

$$x_i = x_L - l_i q_i. \tag{6}$$

2.2 Differential Flatness

A system is differentially flat if there exists a set of flat outputs which can be used to express all the system inputs and states only using the flat outputs and a finite number of higher order derivatives, [1]. The system comprising of the n quadrotors with a cable-suspended point mass load in 3D was shown to be differentially flat, see [8, Lemma 1]. The system we are considering above is a planar system, and as we will establish below, this system is differentially flat as well.

***Lemma* 1.** (Differential-Flatness of the planar n quadrotor point-mass load system $\mathscr{Y}_n = (x_L, T_i q_i)$, *for* $i \in \{2, \cdots, n\}$ *is a set of flat outputs for the planar n quadrotor point-mass load system.*

Proof. From x_L *and its higher order derivatives, the left hand side of* (3) *can be determined. Moreover, substituting the other flat outputs,* $T_i q_i$*, for* $i \in \{2, \cdots, n\}$ *into* (3)*, we can determine* $T_1 q_1$*. The unit vectors* q_i *can be then determined from* $T_i q_i$*. Then, from* (6)*, the quadrotor position* x_i *can be determined. Substituting for* $T_i q_i$ *and the higher-order derivatives of* x_i *in* (1)*, we can determine the thrust vector* $f_i R_i e_2$*, from which the scalar thrust* f_i*, and the quadrotor attitude* ϕ_i *can be determined. Substituting for the higher-order derivatives of* ϕ_i *in* (2) *the torque* τ_i *can be determined.*

Having established the planar n quadrotor system with a point-mass load is differentially-flat, we will next see how to study the problem of determining the minimum and maximum number of quadrotors required for a specific aerial manipulation task.

2.3 Determining Number of Agents Required

We now consider the problem of determining the minimum (maximum) number of agents required for the task of moving a cable-suspended load along a desired trajectory. Given a load trajectory, our goal is to formulate a method to determine the optimum number of agents subject to input constraints such as constraints on thrust / moment, and/or state constraints such as constraints on tension / load attitude / quadrotor attitude, etc. In particular, we define:

Minimum number of quadrotors as the number of quadrotors n_{min} such that each and every quadrotor in the cooperative system exerts a thrust whose magnitude is less than or equal to a maximum thrust F_{max}; and

Maximum number of quadrotors as the number of quadrotors n_{max} at which upon addition of one extra quadrotor, the total thrust needed for traversal of the load trajectory decreases by less than and efficiency parameter η, where we select $\eta = 0.1$.

Our goal then is to find the minimum (maximum) number of quadrotors to have the suspended load follow a desired trajectory. Our approach to solve this problem will make use of the fact that the system is differentially flat, such that we can compute the entire state and input of the system subject to the constraint of the load moving along the desired trajectory.

Given the flat outputs comprising of the load trajectory and $n-1$ cable tension vectors, i.e., given $\mathscr{Y}_n = (x_L, T_i q_i)$, for $i \in \{2, \cdots, n\}$, we know everything in the system. However, since we are only given the load trajectory, x_L, we will make the following simplifying assumptions to obtain a complete set of flat outputs:

A.1. The angle of separation between each cable is fixed and constant at each instant in time along the load trajectory.
A.2. The cables are massless, taut, and are of the same length.
A.3. Desired load trajectories can be expressed as a linear combination of arcs of arbitrary ellipses of varying semi-major and semi-minor axes.

Remark 1. *Note that since we considering the planar cable-suspended aerial manipulation problem, all the quadrotors are coplanar as depicted in Figure 2.*

Remark 2. *The above assumptions place strict requirements on the system. Assumption A.1 enables a simpler choice of the flat outputs for the n quadrotor system that converts a nested optimization problem, where both the number of agents and the flat outputs are optimized sequentially, to an optimization problem where only the number of agents is solved for. Furthermore, to prevent collisions between quadrotors, a minimum separation needs to be enforced, motivating the introduction of ψ.*

Assumption A.2 allows for a simplification of the dynamics of the system. In particular, a cable with (distributed) mass, instead of a massless cable, will convert the equations of motion into a partial differential equation. Moreover, allowing for the cable to switch between being taut and slack requires modeling the system as a hybrid model like was done in [3, 9, 10].

Finally, Assumption A.3 is general and can easily approximate smooth trajectories.

As per the above assumptions, we consider the system of quadrotors to be positioned equidistantly in an arc around the point mass load with a separation angle of 2ψ between neighboring cables, as shown in Figure 2. The angle of each quadrotor with respect to it's cable is then given by

$$\theta_i = -\psi(n-1) + 2(i-1)\psi. \tag{7}$$

The vector sum of all tensions in the cables must be balanced by the forces generated by the quadrotors and the load. This is expressed by

$$\sum_{i=1}^{n} T_i q_i = -m_L(\ddot{x}_L - g e_2). \tag{8}$$

Concurrently the forces acting on each quadrotor can be expressed by

$$T_i q_i = \frac{R(\theta_i)\sum_{i=1}^{n} T_i q_i}{n\cos(\theta_i)}, \tag{9}$$

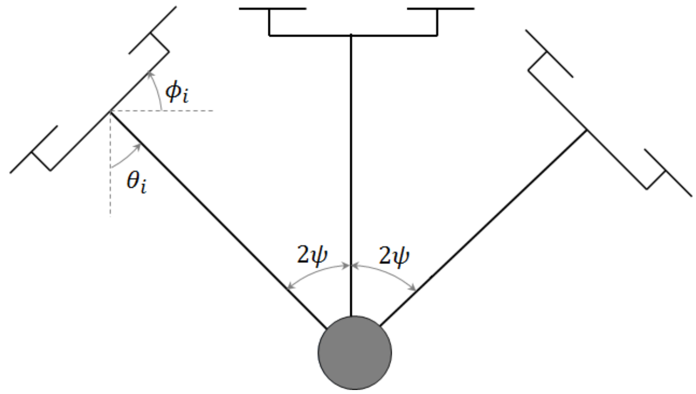

FIGURE 2. 3 quadrotor system with an angle separation of 2ψ. The angle of the i^{th} quadrotor with respect to the horizontal, ϕ_i, as well as with respect to the cable, θ_i are also shown.

where $R(\theta_i)$ is a rotation matrix that rotates points in the xy-Cartesian plane counter-clockwise through an angle θ_i. This then provides a complete set of flat outputs $\mathscr{Y}_n$.

Since any desired trajectory can be approximately broken up into a sum of sinusoidal basis functions, consequently, we express a desired trajectory as a sequence of arcs of ellipses at different constant velocities. Thus, instead of finding the optimal number of agents for an arbitrary trajectory, we focus on finding the optimal number of agents for a constant velocity elliptical section of the trajectory, which can then be extended for the entire trajectory.

To not bias our results due to asymmetry, we consider all quadrotors to have the same mass and able to deliver a thrust bounded at $F_{\max} = 20N$. In the subsequent sections, we make use of differential flatness to analytically and iteratively calculate the thrust of each quadrotor in terms of the desired trajectory, while varying the following variables:

a. Angle of separation of the cables, ψ,
b. Cable length, l,
c. Mass of the quadrotors, m_i,
d. Mass of the load, m_L,
e. Angular velocity of the elliptical trajectory, ω.

3 RESULTS

Having presented the equations of motion, differential flatness, simplifying assumptions for computing the flat outputs, and finally the list of parameters we will vary to find the minimum (maximum) number of agents, we now present some results of our computations. These results will be discussed in detail in the next section.

Our process is to derive the equations of thrust for each individual quadrotor in terms of load mass, quadrotor mass, cable length, time, angle of separation, and angular velocity of the load

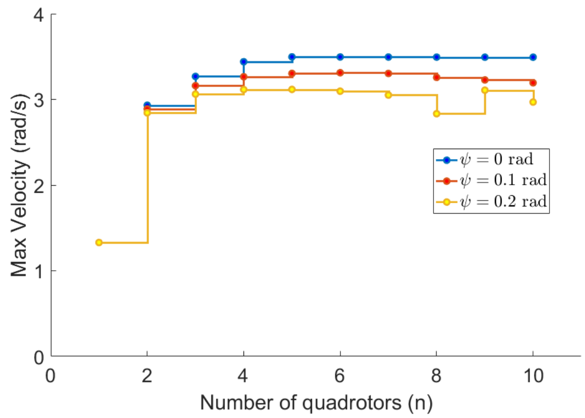

FIGURE 3. The max velocity of the load trajectory versus the number of quadrotors of the system, given a cable length of 1 meter, quadrotor mass of 1 kg, and load mass of 1 kg, with quadrotor separation angles of 0, 0.1, and 0.2 rad respectively. As can be seen, (a) for a given angle of separation, adding more quadrotors does not increase the maximum velocity possible beyond a certain range, and (b) with larger angles of separation, there's a decrease in the maximum velocity possible.

using differential-flatness. (Note that the quadrotor thrust can be expressed as the 4^{th} time-derivative of the load trajectory.) We then find the effect of load mass, quadrotor mass, cable length, and angle of separation on the thrust of the system and maximum velocity.

We define **maximum velocity** to be the largest possible velocity of the load before any of the quadrotors in the system requires applying a thrust more than F_{max} anywhere along the trajectory. In Figures 3, 4, 5 we examine the effect of varying load mass, quadrotor mass, and angle of separation on the maximum velocity for a system of n quadrotors, with increasing values of n.

Next, we examine the effect of different variables on the quadrotor's thrust. To achieve this, we hold the load velocity constant and take the largest thrust along the trajectory as the **minimum thrust** value of F_{max} necessary for having the load track the trajectory. In Figures 6, 7, 8 we examine the effect of varying load mass, quadrotor mass, and angle of separation on the maximum thrust required for a system of n quadrotors, with increasing values of n.

The effect of cable length is also examined. Figure 9 shows the trajectories of a single quadrotor system as cable length is varied. Figure 10 shows the relation between thrust and cable length when all other variables remain constant.

Lastly, we created a random trajectory and evaluated the minimum (maximum) number of quadrotors required, as defined in Section 2.3, for the task of transporting the load along the trajectory at angular velocity of 2 rad/s.

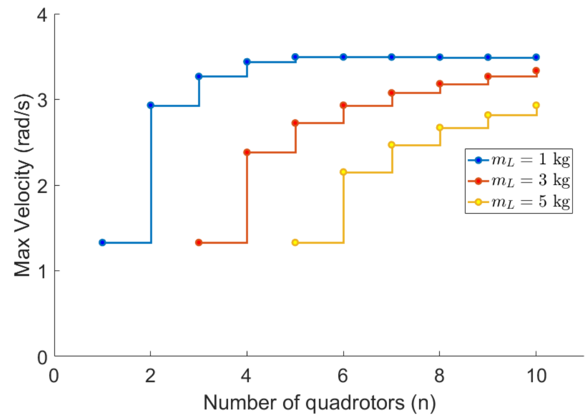

FIGURE 4. The max velocity of the load trajectory versus the number of quadrotors of the system, given a cable length of 1 meter, quadrotor mass of 1 kg, and load mass of 1, 3, and 5 kg with quadrotor separation angle of 0 rad. As can be seen, (a) for a given load mass, adding more quadrotors helps increase the maximum velocity until a certain limit, and (b) increasing the load mass requires increasing number of quadrotors, for instance to achieve 1.3 rad/s requires 1, 3, and 5 quadrotors while to achieve 2.9 rad/s requires 2, 6, 10 quadrotors, for 1, 3, 5 Kg load masses respectively.

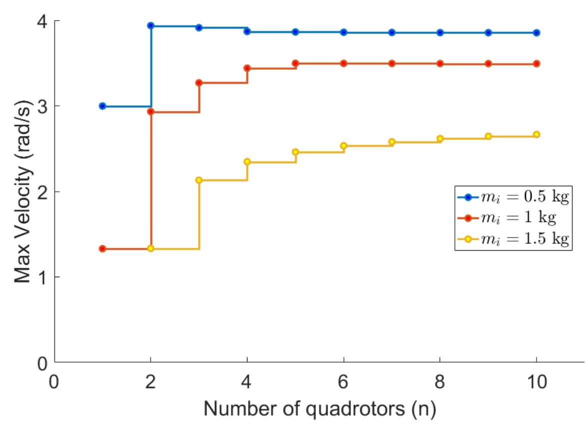

FIGURE 5. The max velocity of the load trajectory versus the number of quadrotors of the system, given a cable length of 3 meters, quadrotor mass of .5, 1, 1.5 kg, and load mass of 1 kg with quadrotor separation angle of 0 rad. As can be seen, (a) for a given quadrotor mass, adding more quadrotors helps to increase the maximum velocity until a certain limit, and (b) increasing the quadrotor mass has a dispproportinate effect on the achievable max velocity, for instance to achieve 2.9 rad/s requires one 0.5 kg, two 1 kg, and possibly very large number of 1.5 kg quadrotors.

4 DISCUSSION

Having presented the results of our computations to see the effect of number of quadrotors, separation angle, load and quadrotor masses on maximum velocity of the load trajectory and minimum thrust required, as well as the effect of cable length

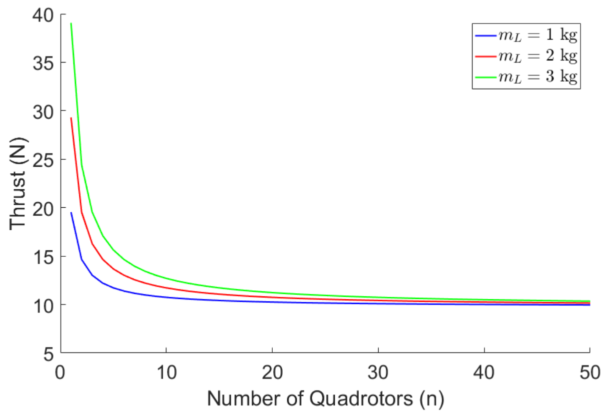

FIGURE 6. The minimum thrust as a function of the number of quadrotors with angle of separation 0 rad, velocity of 1 rad/s, cable length of 1 m, and quadrotor mass of 1kg, with a varying load mass. While increasing the number of quadrotors decreases the minimum thrust required, additionally, in all cases the minimum thrust goes to the same non zero limit asymptotically irrespective of the load mass.

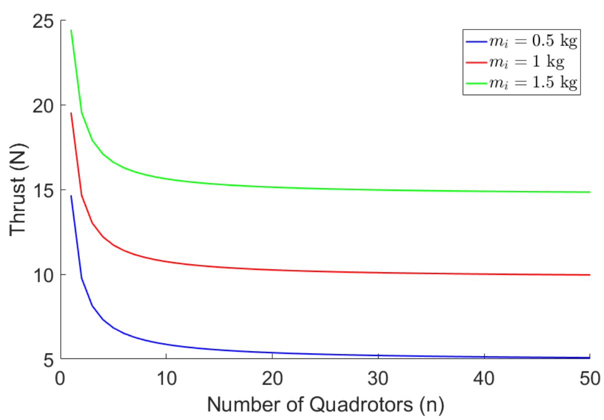

FIGURE 7. The minimum thrust as a function of the number of quadrotors with angle of separation 0 rad, velocity of 1 rad/s, cable length of 1 m, and load mass of 1 kg, with a varying quadrotor mass. While increasing the number of quadrotors decreases the minimum thrust required, changing the quadrotor mass results in the minimum thrust asymptotically converging to a different limit equal to $m_i g$.

on load trajectory and quadrotor thrust, we will now present a detailed discussion on our results.

4.1 Effect of Angle of Separation

We expect that varying the half angle of separation ψ of the quadrotors will have a noticeable effect on the system. In par-

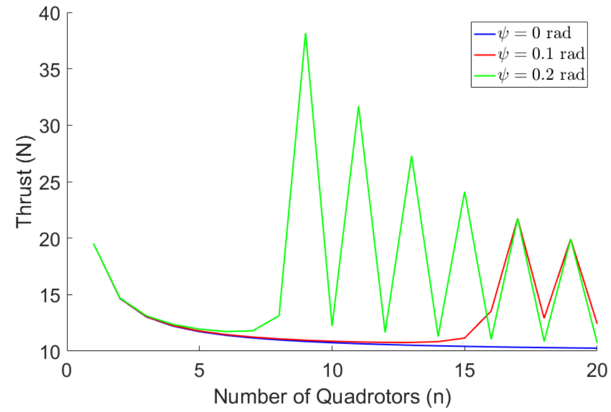

FIGURE 8. The minimum thrust as a function of the number of quadrotors with cable length 1 m, velocity of 1 rad/s and load and quadrotor mass of 1 kg, with a varying angle of separation. As $n\psi \approx \pi/2$, the thrust changes abruptly. This is due to the resulting quadrotor and cable arrangement such that there are a pair of quadrotors pulling horizontally on the load.

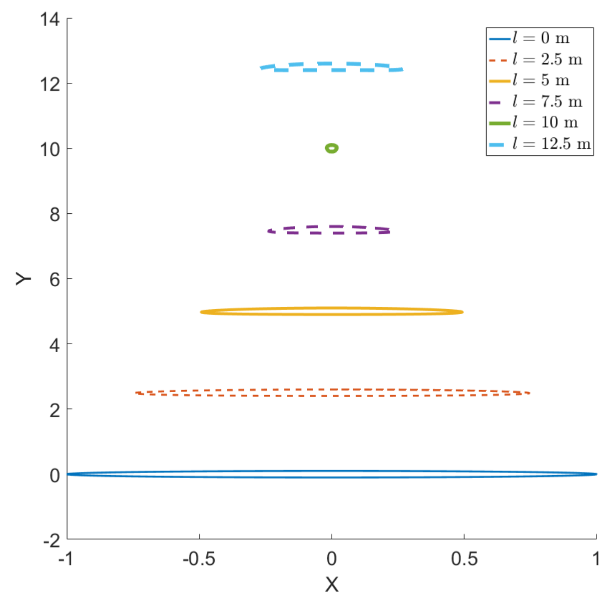

FIGURE 9. Quadrotor trajectories for different cable lengths. A single quadrotor with a suspended load is considered with quadrotor and load masses of $m_1 = m_L = 1kg$ and load trajectory velocity of $\omega = 1rad/s$. The smallest motion of the quadrotor occurs when the cable length is $l = 10m$.

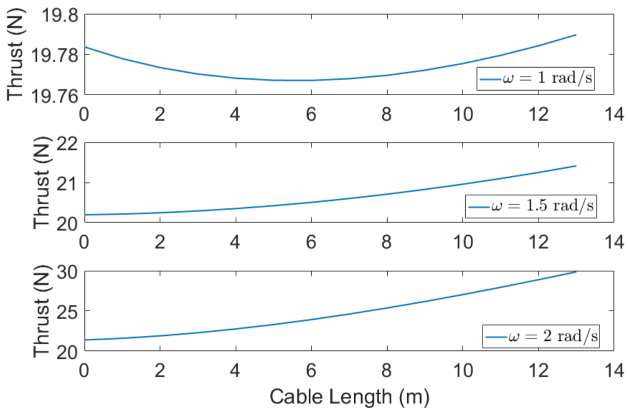

FIGURE 10. The total thrust with a given cable length of a one quadrotor system with a quadrotor mass of 1 kg, separation angle of 0 rad, load mass of 1 kg, and velocity of 1, 1.5, and 2 rad/s.

TABLE 2. Minimum and Maximum number of quadrotors required as a function of angle of separation and maximum angular velocity of the load

Velocity (rad/s)	Angle (rad)	n_{min}	n_{max}
0.5	0	1	4
0.5	0.1	1	4
0.5	0.2	1	4
1	0	1	4
1	0.1	1	4
1	0.2	1	4
1.5	0	1	4
1.5	0.1	1	4
1.5	0.2	1	4
2	0	2	4
2	0.1	2	4
2	0.2	2	4

ticular, it is obvious that having no angle of separation (although impossible to physically realize) is optimal, since each quadrotor can use all of its thrust to follow the given trajectory. Consequently, increasing the angle of separation decreases the maximum velocity as shown in Figure 3. This is because as the angle increases, the norm of the thrust applied in the direction of the acceleration required for the desired trajectory decreases. Additionally when a quadrotor has an angle of close to 90 degrees with respect to the cable ($\theta_i \approx 90°$) the thrust spikes as shown in Figure 8. This is due to the singularity in the denominator of (9). This is also the reason for the dip at $n = 8$ for $\psi = 0.2$ *rad* in Figure 3.

4.2 Effect of Load and Quadrotor Mass

Our results show that varying the mass of the quadrotors has a larger effect on the system than load mass. As the number of quadrotors increase, the ratio between the mass of the quadrotors and the mass of the load increases. As seen in the Figures 4, 5, a small increase of the quadrotor mass decreases the maximum velocity possible more than a small increase of the mass load. Furthermore, as seen in Figures 6, 7, increasing the quadrotor mass also increases the limit of thrust when adding more quadrotors while increasing the load mass does not. This is since more thrust is required for simple hovering as the quadrotor mass increases.

4.3 Effect of Cable Length

We find that cable length has an important effect on the system. We present the results for varying cable lengths in Figure 9. In the case of zero length cable, the load can be assumed to be affixed to the quadrotor. In this case the trajectory of the quadrotor is identical to the trajectory of the load.

As the cable lengths varies from 0 to 12.5m, we notice the quadrotor trajectory becoming smaller with a minimum at cable length of 10 m and an increase thereafter. We estimate that a non-zero cable length is optimal since the moment of inertia of the system remains low, thus allowing for quick aggressive motion of quadrotors. A longer cable length would make the system sluggish since the load would take longer to react to the motion of the quadrotor system.

The relationship between cable length and thrust necessary for a single quadrotor system is nonlinear and has a non-zero minimum as shown in Figure 10. As velocity increases, the optimal cable length gets closer to zero and the cable length has a larger effect on thrust.

4.4 Minimum and Maximum Number of Quadrotors

Intuitively, one could potentially conclude that every additional quadrotor would increase the maximum load velocity. However, our computations show that even under the degenerate case, when the cable separation angle is zero, additional quadrotors give us diminishing returns.

An example for a general trajectory, is tabulated in table 2 with cable length 1 meter and both load and quadrotor mass at 1 kilogram. Results show that at low load velocity, which corresponds to low cable tension, the maximum number of quadrotors is slightly more than the minimum number of quadrotors (as defined in Section 2.3). As velocity, and with it the cable tension, increases, the difference between the minimum and maximum

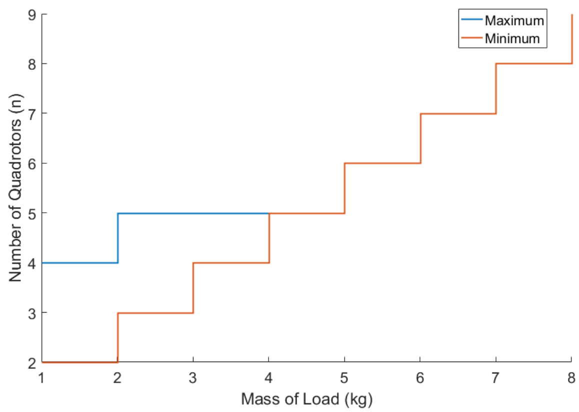

FIGURE 11. The minimum and maximum number of quadrotors versus mass of load in a system with cable length of 1 m, quadrotor mass of 1 kg, and velocity of 2 rad/s.

number of quadrotors gets smaller until they converge. This is because at high cable tensions, additional quadrotors would decrease the useful thrust and add no value to the system. This relationship between the minimum and maximum number of quadrotors is also confirmed by Figure 11, where the maximum and minimum quadrotors are found for a varying load mass, with velocity of the load trajectory of 2 rad/s, cable length of 1m, and quadrotor mass of 1 kg.

5 CONCLUSION

We have proposed a method for optimal selection of the number of quadrotors for dynamically transporting a cable-suspended load along a desired trajectory. We have established that the planar n quadrotor system with a cable suspended load is differentially flat, and presented a methodology to determine the minimum (maximum) number of agents required. We also present results of our computations to study the effect of number of quadrotors, separation angle, load and quadrotor masses on maximum velocity of the load trajectory and minimum thrust required, as well as the effect of cable length on load trajectory and quadrotor thrust. Our results show that increasing the number of quadrotors in the planar case past an optimal threshold produces diminishing returns; increasing agent mass has a larger effect on the system than increasing load mass; and that the length of the cable has a non-trivial effect on the system.

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