Safety-Critical Control for Dynamical Bipedal Walking with Precise Footstep Placement

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Abstract: This paper presents a novel methodology to achieve dynamic walking for underactuated and hybrid dynamical bipedal robots subject to safety-critical position-based constraints. The proposed controller is based on the combination of control Barrier functions and control Lyapunov functions implemented as a state-based online quadratic program to achieve stability under input and state constraints, while simultaneously enforcing safety. The main contribution of this paper is the control design to enable stable dynamical bipedal walking subject to strict safety constraints that arise due to walking over a terrain with randomly generated discrete footholds and overhead obstacles. Evaluation of our proposed control design is presented on a model of RABBIT, a five-link planar underacted bipedal robot with point feet.

Keywords: Safety-critical; Nonlinear control; Lyapunov function; Quadratic programming.

1. INTRODUCTION

Cyber physical systems (CPSs) have strict safety constraints and designing controllers that provide formal guarantees of enforcing these safety constraints is critical. In this paper we consider a specific CPS, a bipedal robot. Bipedal walking is subject to several safety constraints. One that is most visual is that of walking over discrete footholds, requiring critical guarantees of precise foot placements for ensuring the safety of the bipedal robot. Here we consider the problem of dynamic walking, and develop a controller that provides guarantees on stability and safety. The proposed methodology is also applicable to other CPSs whose dynamics evolve on complex nonlinear manifolds as illustrated in Wu and Sreenath (2015).

Footstep placement for fully actuated legged robots has been carried out by several researchers Kajita et al. (2003); Kuffner et al. (2001); Chestnutt et al. (2005). Impressive results in footstep planning and placements in obstacle filled environments with vision-based sensing is carried out in Michel et al. (2005); Chestnutt et al. (2003). However, these methods essentially rely on quasi-static walking using the ZMP criterion which only enables slow walking with small steps. Moreover, for a point-foot legged robot, the above methods are not feasible. The DARPA Robotics Challenge has inspired several new methods, some based on mixed-integer quadratic programs Deits and Tedrake (2014). However, as mentioned in (Deits, 2014, Chap. 4), the proposed method of mixed-integer based footstep planning does not offer dynamic feasibility even for a simplified model. On the other hand, footstep placement for dynamic locomotion typically rely on simple models with massless legs, see Desai and Geyer (2012); Rutschmann et al. (2012); Wu and Geyer (2013). Although these results are impressive, extending them to non-trivial bipedal models is hard. Recent results of using centroidal dynamics for whole-body dynamic motion including arm and foot contacts in Dai et al. (2014) is impressive, however stability guarantees of the hybrid locomotion is unclear.

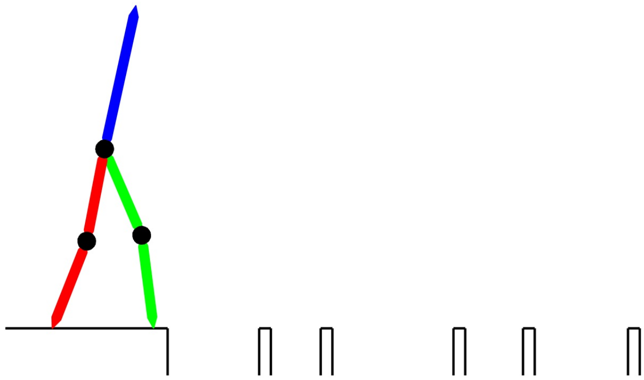

Fig. 1. The problem of dynamically walking over a randomly generated set of discrete footholds. The discrete footholds serve as strict safety constraints that need to be enforced with formal guarantees for the safety of the bipedal robot. Although the proposed safety-critical control is developed in this paper for this particular problem, it's also applicable to more general CPSs.

The proposed approach in this paper deviates from these prior results by starting off with a periodic dynamic walk-

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ing gait with formal stability guarantees and enforcing safety-critical constraints for achieving precise footstep placement. The proposed controller can also enforce various input and state-based constraints. It must be noted that precise footstep placement for dynamic walking on underactued bipedal robots with nonlinear and hybrid dynamics with provable stability and safety is challenging. Our research develops on recent work on control Lyapunov function based quadratic programs for bipedal robots and control Barrier function based quadratic programs for autonomous cruise control in cars, see Galloway et al. (2015); Ames et al. (2014a). The proposed methodology enables guaranteeing the trajectory of the swing foot to be within certain constraints, so as to result in a given step length when the swing foot hits the ground. This results in placing the foot precisely on the discrete foothold.

In particular, we employ the method of Hybrid Zero Dynamics (HZD) and virtual constraints, Westervelt et al. (2003, 2007), which has been successful in dealing with the hybrid and underactuated dynamics of legged locomotion. Experimental implementations of the HZD method using input-output linearization with PD control for dynamic walking has been carried out in Sreenath et al. (2011) and dynamic running in Sreenath et al. (2013) on MABEL. Recent work on control Lyapunov function (CLF)-based controllers has resulted in HZD-based stable walking, Ames et al. (2014b). The flexible control design using CLFs has been carried out through online quadratic programs (QPs), facilitating incorporating additional constraints into the control computation, Galloway et al. (2015), and L_1 adaptive control with model uncertainty in Nguyen and Sreenath (2015a).

Although the HZD based control design for dynamic legged locomotion is powerful, these types of controllers cannot directly help us to enforce safety-critical constraints, such as walking over discrete footholds and walking under obstacles. One way of enforcing the safety constraints is to recompute the walking gait by changing the virtual constraints used for walking. Virtual constraints are a set of output functions that need to be regulated by the controller to achieve periodic walking. However, this involves a complex nonlinear constrained optimization which is time-consuming and intractable for online implementation in a feedback controller.

The main contribution of this paper is a novel safety-critical control strategy that can guarantee precise footstep placement for dynamic walking of a hybrid, nonlinear, underactuated bipedal system. We do this by combining control Lyapunov function based quadratic programs (CLF-QPs) Galloway et al. (2015) for tracking the nominal virtual constraints while respecting the saturation limits of control inputs, and control Barrier function based quadratic programs (CBF-QPs) Ames et al. (2014a) to guarantee state dependent safety constraints. The goal of this paper is to relax the tracking behavior of the nominal gait by enforcing a set of state dependent safety constraints, governed by control Barrier functions, that guide the swing foot trajectory to the discrete foothold and bend the torso to avoid overhead obstacles. Our method enables dealing with a large range of desired foothold separations with precise placement of footsteps on small footholds that are less than 5% of the leg length. As noted earlier, this work is generalizable to other CPSs (see Wu and Sreenath (2015).)

The rest of the paper is organized as follows. Section 2 revisits control Lyapunov function-based quadratic programs (CLF-QPs). Section 3 revisits Control Barrier Function and its combination with CLF-QP. Section 4 presents the proposed CBF-CLF-QP based feedback controller for enforcing safety constraints. Here we will first present its application to the simpler problem of avoiding overhead obstacles before developing for the main problem of dynamic bipedal walking over a terrain with discrete footholds (see Fig. 1). Section 5 then presents numerical validation of the controller on the model of RABBIT, a five-link planar bipedal robot. Finally, Section 6 provides concluding remarks.

2. CONTROL LYAPUNOV FUNCTION BASED QUADRATIC PROGRAMS REVISITED

In this section we start by introducing a hybrid dynamical model that captures the dynamics of a bipedal robot. We then review recent innovations on control Lyapunov functions for hybrid systems and control Lyapunov function based quadratic programs, introduced in Ames et al. (2014b) and Galloway et al. (2015) respectively.

2.1 Model

This paper will focus on the specific problem of walking of bipedal robots such as RABBIT (described in Chevallereau et al. (2003)), which is characterized by single-support continuous-time dynamics, when one foot is assumed to be in contact with the ground, and double-support discrete-time impact dynamics, when the swing foot undergoes an instantaneous impact with the ground. Such a hybrid model is obtained as,

$$\mathcal{H} = \begin{cases} \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = f(q,\dot{q}) + g(q,\dot{q})u, & (q^-,\dot{q}^-) \notin S, \\ \\ \begin{bmatrix} q^+ \\ \dot{q}^+ \end{bmatrix} = \Delta(q^-,\dot{q}^-), & (q^-,\dot{q}^-) \in S, \end{cases} \tag{1}$$

where $q \in \mathcal{Q}$ is the robot's configuration variables, $u \in \mathbb{R}^m$ is the control inputs, representing the motor torques, $(q^-, \dot{q}^-)$ represents the state before impact and $(q^+, \dot{q}^+)$ represents the state after impact, S represents the switching surface when the swing leg contacts the ground, and Δ represents the discrete-time impact map.

We also define output functions $y \in \mathbb{R}^m$ of the form

$$y(q) := H_0 q - y_d(\theta(q)), \tag{2}$$

where $\theta(q)$ is a strictly monotonic function of the configuration variable q, H_0 is an appropriately-sized matrix prescribing linear combinations of state variables to be controlled, and $y_d(\cdot)$ prescribes the desired evolution of these quantities (see Sreenath et al. (2011) for details.) The method of Hybrid Zero Dynamics (HZD) aims to drive these output functions (and their first derivatives) to zero, thereby imposing "virtual constraints" such that the system evolves on the lower-dimensional zero dynamics manifold, given by

$$Z = \{(q,\dot{q}) \in T\mathcal{Q} \mid y(q) = 0,\ L_f y(q,\dot{q}) = 0\}, \tag{3}$$

where L_f denotes the Lie derivative, Isidori (1989). In particular, the dynamics of the system $\mathcal{H}$ in (1) restricted to Z, given by $\mathcal{H}|_Z$, is the underactuated dynamics of the system and is forward-invariant. Periodic motion such as walking is then a hybrid periodic orbit $\mathscr{O}$ in the statespace with $\mathscr{O}_Z$ being it's restriction to Z.

2.2 Input-output linearization

If $y(q)$ has vector relative degree 2, then the second derivative takes the form

$$\ddot{y} = L_f^2 y(q,\dot{q}) + L_g L_f y(q,\dot{q})\ u. \tag{4}$$

We can then apply the following pre-control law

$$u(q,\dot{q}) = u^*(q,\dot{q}) + (L_g L_f y(q,\dot{q}))^{-1}\ \mu, \tag{5}$$

where

$$u^*(q,\dot{q}) := -(L_g L_f y(q,\dot{q}))^{-1} L_f^2 y(q,\dot{q}), \tag{6}$$

and μ is a stabilizing control to be chosen. Defining transverse variables $\eta = [y,\ \dot{y}]^T$, and using the IO linearization controller above with the pre-control law (5), we have,

$$\ddot{y} = \mu, \tag{7}$$

$$\implies \begin{bmatrix}\dot{y}\\ \ddot{y}\end{bmatrix} = \dot{\eta} = \begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}\eta + \begin{bmatrix}0\\ I\end{bmatrix}\mu. \tag{8}$$

The closed-loop dynamics can be expressed in terms of the transverse variables η and the states $z \in Z$ from (3) (instead of in terms of the state $(q,\dot{q})$), to take the form,

$$\begin{aligned}\dot{\eta} &= \bar{f}(\eta,z) + \bar{g}(\eta,z)\mu, \\ \dot{z} &= p(\eta,z),\end{aligned} \tag{9}$$

where $\bar{f}(\eta,z) = F\eta$ and $\bar{g}(\eta,z) = G$, with

$$F = \begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}, \qquad G = \begin{bmatrix}0\\ I\end{bmatrix}, \tag{10}$$

and $p(\eta,z)$ arises from the mapping between $(q,\dot{q})$ and (η,z).

2.3 CLF-based Quadratic Programs

A control approach based on control Lyapunov functions, introduced in Ames et al. (2014b), provides guarantees of exponential stability for the traverse variables η. In particular, a function $V(\eta)$ is a *exponentially stabilizing control Lyapunov function (ES-CLF)* for the system (9) if there exist positive constants $c_1, c_2, \lambda > 0$ such that

$$c_1\|\eta\|^2 \le V(\eta) \le c_2\|\eta\|^2, \tag{11}$$

$$\dot{V}(\eta,\mu) + \lambda V(\eta) \le 0. \tag{12}$$

In our problem, we chose a CLF candidate as follows

$$V(\eta) = \eta^T P \eta. \tag{13}$$

The time derivative of the CLF (13) is computed as

$$\dot{V}(\eta,\mu) = L_{\bar{f}}V(\eta) + L_{\bar{g}}V(\eta)\mu, \tag{14}$$

where

$$\begin{aligned}L_{\bar{f}}V(\eta) &= \eta^T(F^T P + PF)\eta, \\ L_{\bar{g}}V(\eta) &= 2\eta^T P G.\end{aligned} \tag{15}$$

The CLF condition takes the form

$$L_{\bar{f}}V(\eta) + L_{\bar{g}}V(\eta)\mu + \lambda V(\eta) \le 0. \tag{16}$$

If this inequality holds, then it implies that the output η will be exponentially driven to zero by the controller. The CLF-QP based controller presented in Galloway et al. (2015) takes the form:

$$\begin{aligned}\mu^* = &\underset{\mu,d_1}{\operatorname{argmin}} \quad \mu^T\mu + p_1\ d_1^2 \\ &\text{s.t.} \quad \dot{V}(\eta,\mu) + \lambda V(\eta) \le d_1 \qquad \textbf{(CLF)} \\ &\qquad\ \ A_{AC}(q,\dot{q})\ \mu \le b_{AC}(q,\dot{q}) \quad \textbf{(Constraints)}\end{aligned} \tag{17}$$

where p_1 is a large positive number that represents the penalty of relaxing the CLF condition (12) and $A_{AC},\ b_{AC}$ represent additional constraints such as torque constraints, contact force constraints, friction constraints and joint limit constraints. In order to incorporate the additional constraints, we need to relax the CLF condition (12) to guarantee the feasibility of the optimization problem. Therefore, this type of relaxed CLF-QP potentially cannot ensure the same type of stability claims as those provided by (Ames et al., 2014b, Thm. 2), as the relaxations of the inequality constraint on the time-derivative $\dot{V}(\eta)$ result in a loss of the CLF quality for $V(\eta)$. However, under appropriate conditions, we still retain the stability of the hybrid periodic orbit, see Nguyen and Sreenath (2015b) for further details.

We can also represent this in a standard quadratic program form:

CLF-QP:

$$\begin{aligned}&\underset{\mathbf{u}=\begin{bmatrix}\mu\\ d_1\end{bmatrix}}{\operatorname{argmin}} \quad \frac{1}{2}\mathbf{u}^T H \mathbf{u} \\ &\text{s.t.} \quad A_{CLF}\mathbf{u} \le b_{CLF} \qquad \textbf{(CLF)} \\ &\qquad\ \ A_{AC}\ \mathbf{u} \le b_{AC} \qquad \textbf{(Additional Constraints)}\end{aligned} \tag{18}$$

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where

$$H = \begin{bmatrix}I & 0\\ 0 & p_1\end{bmatrix}; \tag{19}$$

$$A_{CLF} = [L_{\bar{g}}V \ \ {-1}]\,; b_{CLF} = -L_{\bar{f}}V - \lambda V.$$

If the additional constraints above are constraints on the control inputs, such as $u_{min} \le u \le u_{max}$, then we have $A_{AC} = A_{TS},\ b_{AC} = b_{TS}$, where,

$$A_{TS} = \begin{bmatrix}(L_g L_f y(q,\dot{q}))^{-1} & 0\\ -(L_g L_f y(q,\dot{q}))^{-1} & 0\end{bmatrix}; b_{TS} = \begin{bmatrix}u_{max} - u^*\\ u^* - u_{min}\end{bmatrix}. \tag{20}$$

The above torque constraints are due to the definition of the precontrol in (5). This formulation opened a novel method to guarantee stability of the nonlinear systems with respect to additional constraints, such as torque saturation in Galloway et al. (2015) and L_1 adaptive control in Nguyen and Sreenath (2015a).

Having revisited control Lyapunov function based quadratic programs, we will next revisit control Barrier functions and control Barrier function based quadratic programs.

3. CONTROL BARRIER FUNCTION REVISITED

3.1 Control Barrier Function

Consider an affine control system:

$$\dot{x} = f(x) + g(x)u \tag{21}$$

with the goal to design a controller to keep the state x in the set

$$\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\} \tag{22}$$

where $h : \mathbb{R}^n \to \mathbb{R}$ is a continuously differentiable function. Then, a function $B : \mathcal{C} \to \mathbb{R}$ is a Control Barrier Function (CBF), Ames et al. (2014a), if there exists class $\mathcal{K}$ function α_1 and α_2 such that, for all $x \in Int(\mathcal{C}) = \{x \in \mathbb{R}^n : h(x) > 0\}$,

$$\frac{1}{\alpha_1(h(x))} \leq B(x) \leq \frac{1}{\alpha_2(h(x))}, \tag{23}$$

$$\dot{B}(x,u) = L_f B(x) + L_g B(x) u \leq \frac{\gamma}{B(x)}. \tag{24}$$

From Ames et al. (2014a), the important properties of the CBF condition in (24) is that if there exists a Control Barrier Function, $B : \mathcal{C} \to \mathbb{R}$, then $\mathcal{C}$ is forward invariant or in other words, if $x(0) = x_0 \in \mathcal{C}$, i.e., $h(x_0) \geq 0$, then $x = x(t) \in \mathcal{C}, \forall t$, i.e., $h(x(t)) \geq 0, \forall t$. Note that, as mentioned in Ames et al. (2014a), this notion of a CBF is more stricter than standard notions of CBFs in prior literature that only require $\dot{B} \leq 0$.

In this paper, we will use the following Control Barrier Candidate Function:

$$B(x) = \frac{1}{h(x)}. \tag{25}$$

3.2 Combination of CLF and CBF-QP

We define a state dependent constraint $h = h(x) \geq 0$ where $h(x)$ is a real function with relative degree one, i.e,

$$\dot{h}(x,u) = L_f h + L_g h \; u, \tag{26}$$

where $L_g h \neq 0$.

Since in the CLF-QP (18), the controller u is constructed by I-0 linearization (5), in order to combine with the CBF condition, we will rewrite (26) in terms of the variable μ

$$\dot{h}(x,\mu) = L_f h + L_g h(u^* + L_g L_f y^{-1} \mu). \tag{27}$$

Consider a Control Barrier Candidate Function (25), then

$$\dot{B}(x,\mu) = -\frac{\dot{h}(x,\mu)}{h^2(x)} = L_{\bar{f}} B(x) + L_{\bar{g}} B(x) \mu, \tag{28}$$

where,

$$L_{\bar{f}} B(x) = -\frac{1}{h^2(x)}(L_f h(x) + L_g h(x) u^*), \tag{29}$$

$$L_{\bar{g}} B(x) = -\frac{1}{h^2(x)} L_g h(x)(L_g L_f y^{-1}).$$

The CBF condition then is

$$\dot{B}(x,\mu) \leq \frac{\gamma}{B(x)}. \tag{30}$$

We have the following CBF-CLF-QP based controller,

$$\begin{aligned} \mu^* = & \underset{\mu, d_1}{\mathrm{argmin}} \quad \mu^T \mu + p_1 \; d_1^2 \\ & \text{s.t.} \quad \dot{V}(\eta,\mu) + \lambda V(\eta) \leq d_1 \quad \textbf{(CLF)} \\ & \qquad \dot{B}(x,\mu) - \frac{\gamma}{B(x)} \leq 0 \quad \textbf{(CBF)} \\ & \qquad u_{min} \leq u \leq u_{max} \quad \textbf{(TS)} \end{aligned} \tag{31}$$

This can be represented in a standard quadratic program formulation:

CBF-CLF-QP with Torque Saturation:

$$\begin{aligned} & \underset{\mathbf{u}=\begin{bmatrix}\mu \\ d_1\end{bmatrix}}{\mathrm{argmin}} \quad \frac{1}{2}\mathbf{u}^T H \mathbf{u} \\ & \text{s.t.} \quad A_{CLF}\mathbf{u} \leq b_{CLF} \quad \textbf{(CLF)} \\ & \qquad A_{CBF}\mathbf{u} \leq b_{CBF} \quad \textbf{(CBF)} \\ & \qquad A_{TS}\;\mathbf{u} \leq b_{TS} \quad \textbf{(Torque Saturation)} \end{aligned} \tag{32}$$

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where H, A_{CLF}, b_{CLF} are from (19), A_{TS}, b_{TS} from (20), and

$$A_{CBF} = [L_{\bar{g}} B \;\; 0]\,; \quad b_{CBF} = -L_{\bar{f}} B + \frac{\gamma}{B}. \tag{33}$$

Having revisited control Barrier function based quadratic programs, we will now formulate our controller to achieve dynamic walking subject to critical safety constraints such as avoiding overhead obstacles and achieving precise footstep placements on discrete footholds.

4. SAFETY-CRITICAL CONTROL FOR DYNAMICAL BIPEDAL WALKING

4.1 Modification of CBF for position based constraints

As presented in Ames et al. (2014a), the standard CBF is for velocity based constraint, i.e, $h(q,\dot{q}) \geq 0$ with relative degree one. For application of CBF to mechanical systems in general and to the bipedal robotics system in particular, we need to consider position based constraints, i.e, functions of the form $g_b(q) \geq 0$ ("b" is for "Barrier") with relative degree two. The modification of CBF with position based constraint is first mentioned in Wu and Sreenath (2015) as a safety criteria for dynamical systems on manifolds. The idea is that, in order to guarantee the condition $g_b(q) \geq 0$, we construct a barrier constraint with relative degree one:

$$h_{CBF}(q,\dot{q}) = \gamma_b g_b(q) + \dot{g}_b(q,\dot{q}) \geq 0. \tag{34}$$

From this condition, we can guarantee that if $g_b(q)$ starts with a non-negative initial condition $g_b(q_0) \geq 0$ and the constant γ_b is strictly positive, then the condition $h_{CBF} \geq 0$ will guarantee that $g_b(q)$ will never cross zero. Because, if $g_b(q)$ crosses zero from 0^+ to 0^-, it means that:

$$\begin{aligned} & g_b(q) = 0; \dot{g}_b(q,\dot{q}) < 0, \\ \Longrightarrow\; & h_{CBF}(q,\dot{q}) = \gamma_b g_b(q) + \dot{g}_b(q,\dot{q}) < 0, \end{aligned}$$

which violates the CBF condition in (34).

In other words, we can maintain $g_b(q) \geq 0$ by barrier constraint (34). Based on this modification, we now can apply the CBF-CLF-QP based controller (32) presented in Section 3.2, with the barrier function formed by $h = h_{CBF}(q,\dot{q})$ from (34).

We next present application of this approach for bipedal robotics walking with different additional constraints and criteria.

4.2 Avoiding overhead obstacles during walking

We first illustrate the method for the problem of avoiding an overhead low ceiling, denoted as (C), by limiting the

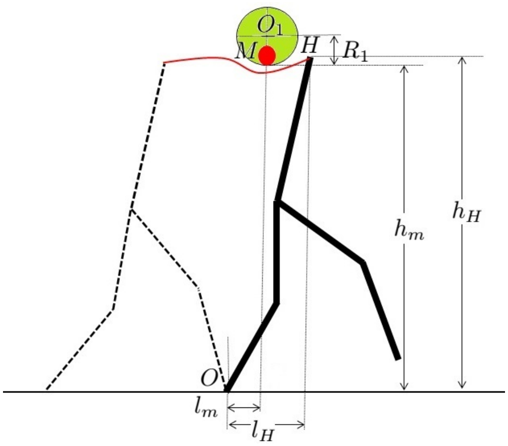


Fig. 2. Geometric explanation of CBF constraints for the problem of avoiding an overhead obstacle located at coordinates (l_m, h_m) with respect to the stance foot. If we can guarantee the trajectory of the robot head H (the red line) to be limited outside the green domain, we can ensure the biped avoids the obstacle (small red circle).

head height of the robot during a walking step. The problem is that there is a low ceiling at a distance h_r from the ground at the position of the i^{th} walking step. We will formulate and apply a CBF in the i^{th} step to guarantee the head height of the robot, h_H, will be always lower than the overhead ceiling ($h_H(q) \le h_r$) so that the robot is able to clear the low ceiling obstacle. Therefore, we just need to guarantee the following position constraint (with relative degree two) during the whole i^{th} step:

$$g_C(q) = h_r - h_H(q) \ge 0. \tag{35}$$

This corresponds to the head height being below the ceiling. The barrier constraint with relative degree 1:

$$h_C(q, \dot{q}) = \gamma_b g_C(q) + \dot{g}_C(q, \dot{q}) \ge 0. \tag{36}$$

The CBF-CLF-QP from (31) can then be used with the Barrier function B as defined in (25) with $h = h_C$ from above.

In the above constraint, the horizontal position of the head l_H, did not matter. For a better construction for this problem, we can also consider avoiding an overhead obstacle (O) at a specific location, as presented in Fig. 2. Here, the red circle with center M represents the overhead obstacle located at coordinates (l_m, h_m) with respect to the stance foot, and the green circle with center O_1 represents the region that the position of the robot head should avoid to ensure no collision with the obstacle. The geometric constraints in the figure can be mathematically represented as follows:

$$O_1H = \sqrt{(R_1 + h_m - h_H)^2 + (l_H - l_m)^2} \ge R_1, \tag{37}$$

Then, the following position constraint,

$$g_O(q) = \sqrt{(R_1 + h_m - h_H)^2 + (l_H - l_m)^2} - R_1 \ge 0, \tag{38}$$

corresponds to the head being outside the green circle with center O_1 in Fig. 2.

The CLF-CBF-QP for this approach will totally be the same as mentioned above, but the simple constraint (35) now becomes more complex as (38).

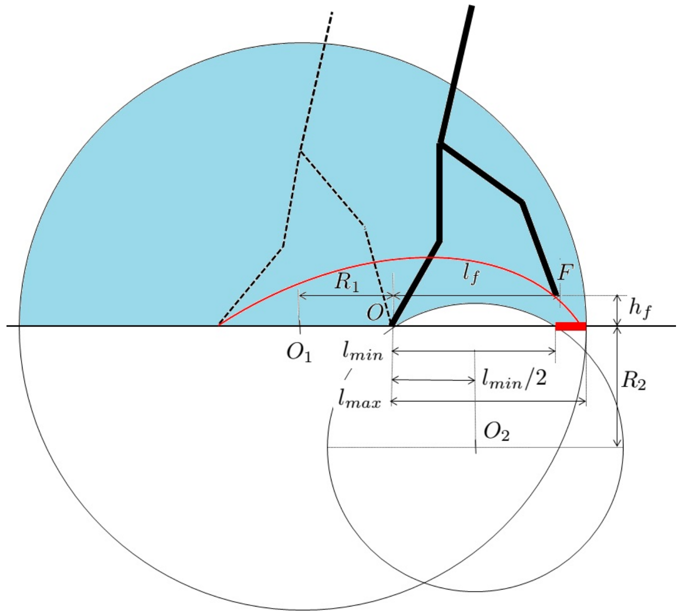


Fig. 3. Geometric explanation of CBF constraints for the problem of bipedal walking over discrete footholds. If we can guarantee the trajectory of the swing foot F (the red line) to be limited in the blue domain, we will force our robot to step onto a discrete foothold position (thick red range on the ground). This approach therefore also provides a safety guarantee against foot scuffing or swing foot being always above the ground prior to contact.

Based on these problems of avoiding overhead obstacles with a single barrier constraint, we can now further develop a control strategy for the problem of footstep placement. One interesting application is the problem of walking over a set of discrete footholds.

4.3 Problem of Walking over Discrete Footholds

If we want to force the robot to step onto a specific position (see Fig. 1), we need to guarantee that the step length when the robot swing foot hits the ground is bounded within a given range $[l_{min}; l_{max}]$. Let $h_f(q)$ be the height of the swing foot to the ground and $l_f(q)$ be the distance between the stance and swing feet. We define the step length at impact as,

$$l_s := l_f(q)|_{h_f(q)=0, \dot{h}_f(q,\dot{q})<0}. \tag{39}$$

The discrete foothold constraint to be enforced then becomes,

$$l_{min} \le l_s \le l_{max}. \tag{40}$$

However, in order to guarantee this final impact-time constraint, we construct a state-based constraint for the evolution of the swing foot during the whole step, so that at impact the swing foot satisfies the discrete foothold constraint (40). We now offer a solution for this issue. The geometric explanation for this is presented in Fig. 3. If we can guarantee the trajectory of the swing foot, F, to be bounded between the domain of the two circles O_1 and O_2, it will imply that the step length when the swing foot hits the ground is bounded within $[l_{min}; l_{max}]$. Mathematical representation for these constraints is stated as follows:

$$O_1F = \sqrt{(R_1 + l_f)^2 + h_f^2} \leq R_1 + l_{max},$$
$$O_2F = \sqrt{(R_2 + h_f)^2 + (l_f - \frac{l_{min}}{2})^2} \geq \sqrt{R_2^2 + (\frac{l_{min}}{2})^2}.$$

When the swing foot hits the ground at the end of the step, $h_f = 0, \dot{h}_f < 0$, the step length is l_s, and therefore,

$$\sqrt{(R_1 + l_s)^2} \leq R_1 + l_{max},$$
$$\sqrt{R_2^2 + (l_s - \frac{l_{min}}{2})^2} \geq \sqrt{R_2^2 + (\frac{l_{min}}{2})^2}. \tag{41}$$

This then implies the discrete foothold constraint (40).

We now define the two barrier constraints based on this approach, through the position constraints

$$g_{ST1}(q) = R_1 + l_{max} - \sqrt{(R_1 + l_f(q))^2 + h_f(q)^2} \geq 0,$$
$$g_{ST2}(q) = \sqrt{(R_2 + h_f)^2 + (l_f - \frac{l_{min}}{2})^2} - \sqrt{R_2^2 + (\frac{l_{min}}{2})^2} \geq 0, \tag{42}$$

to obtain

$$h_{ST_1}(q, \dot{q}) = \gamma_b g_{ST_1}(q) + \dot{g}_{ST_1}(q, \dot{q}) \geq 0,$$
$$h_{ST_2}(q, \dot{q}) = \gamma_b g_{ST_2}(q) + \dot{g}_{ST_2}(q, \dot{q}) \geq 0. \tag{43}$$

The Control Barrier Candidate Functions then are

$$B_1(q, \dot{q}) = \frac{1}{h_{ST_1}(q, \dot{q})}; B_2(q, \dot{q}) = \frac{1}{h_{ST_2}(q, \dot{q})}. \tag{44}$$

We now can apply the CBF-CLF-QP based controller (32) with H, A_{CLF}, b_{CLF} from (19), A_{TS}, b_{TS} from (20) and

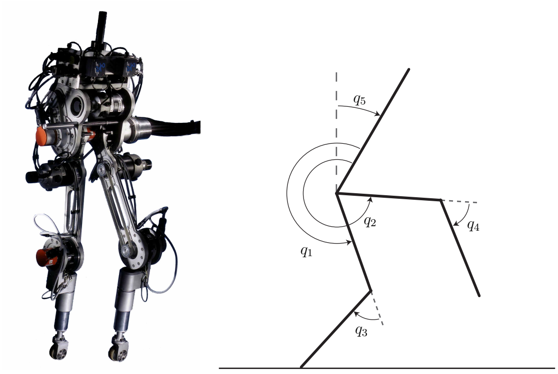
$$A_{CBF} = \begin{bmatrix} L_{\bar{g}}B_1 & 0 \\ L_{\bar{g}}B_2 & 0 \end{bmatrix}; \; b_{CBF} = \begin{bmatrix} -L_{\bar{f}}B_1 + \frac{\gamma}{B_1} \\ -L_{\bar{f}}B_2 + \frac{\gamma}{B_2} \end{bmatrix}. \tag{45}$$

Remark 1. We note that the two problems of avoiding overhead obstacles and walking over discrete footholds, presented here, are just two examples for the application of the CLF-CBF-QP for bipedal robotic walking. With this methodology, and different design for barrier constraints, we can further apply this approach for a variety of additional constraints and criteria during walking control. We can also increase the performance by designing a better barrier constraint.

5. NUMERICAL VALIDATION OF SAFETY-CRITICAL CONTROL FOR DYNAMIC BIPEDAL WALKING

To demonstrate the effectiveness of the proposed CLF-CBF-QP based controller, we will conduct numerical simulations on the model of RABBIT (shown in Figure 4), a planar five-link bipedal robot with a torso and two legs with revolute knees. RABBIT has four actuators to control hip and knee angles, and is connected to a rotating boom which constrains the robot to walk in a circle, approximating planar motion in the sagittal plane. Detailed descriptions of RABBIT and the associated mathematical model can be found in Chevallereau et al. (2003); Westervelt et al. (2004). Fundamental issues in dynamic walking and running on RABBIT can be found in Westervelt et al. (2004) and Morris et al. (2006).

Note that our model of RABBIT that we consider here is the most detailed model of the experimental system,



(a) RABBIT (b) Coordinate system

Fig. 4. (a) RABBIT, a planar five-link bipedal robot with nonlinear, hybrid and underactuated dynamics. (b) The associated generalized coordinate system used, where q_1, q_2 are the relative stance and swing leg femur angles referenced to the torso, q_3, q_4 are the relative stance and swing leg knee angles, and q_5 is the absolute torso angle in the world frame.

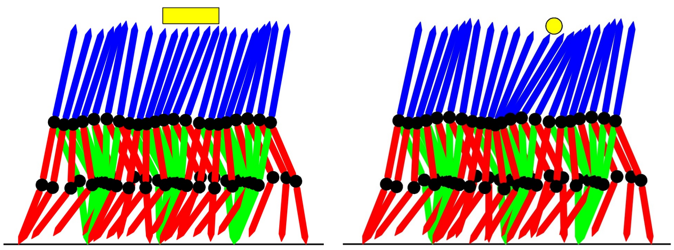


Fig. 5. Simulation of 3 steps of RABBIT walking while avoiding overhead obstacles. The left figure is for the simpler constraint (35) that reduces the head height during the whole step, with the yellow rectangle representing the low ceiling. The right figure is for the more involved constraint (38) to avoid an overhead obstacle at a specific location, with the yellow circle representing the location of the obstacle.

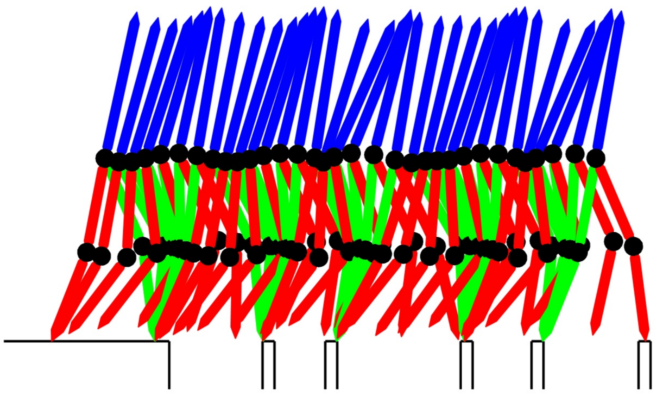


Fig. 7. Simulation of RABBIT walking over a set of five discrete footholds. The black pillars are locations that the robot needs to step onto, and depict the location and size of the discrete footholds. (Video available at `http://youtu.be/AN-nSHsRLEo`)

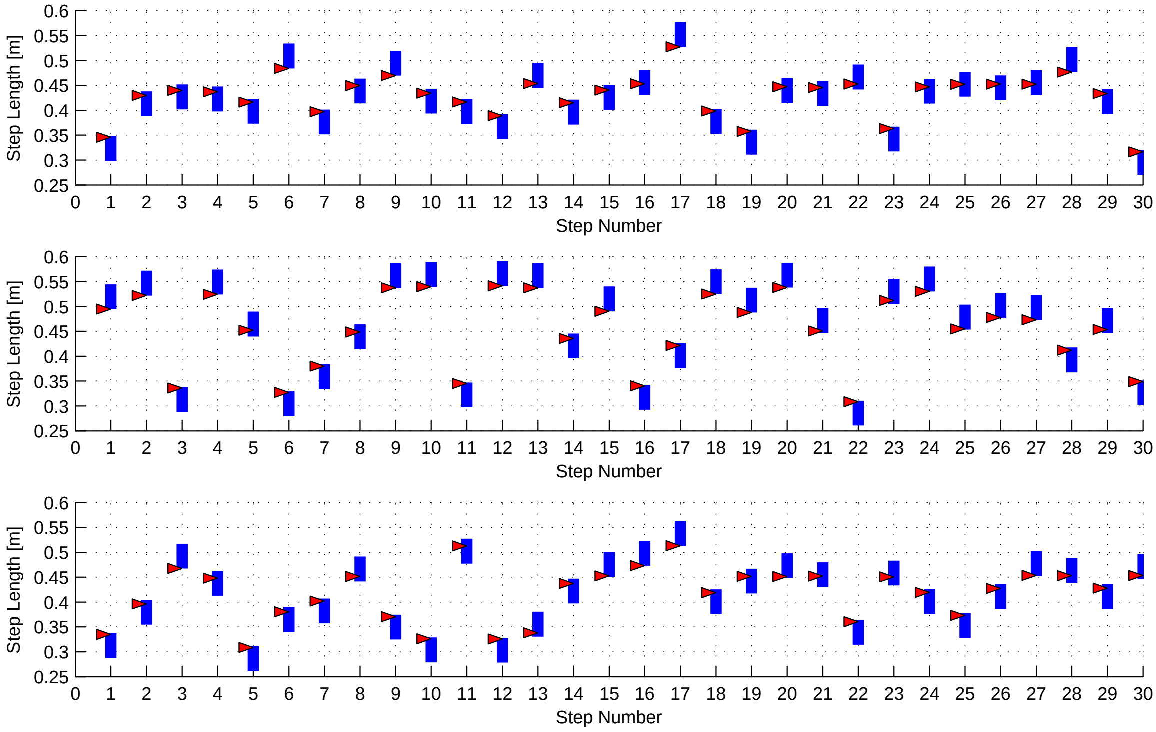


Fig. 6. Three sample runs of RABBIT walking on a set of 30 randomly generated discrete footholds. The desired step lengths, indicating the distance between the footholds, are chosen randomly in the range $0.25m$ to $0.6m$. Red arrows indicate the robot's resulting step length l_s, and represent the placement of the foot. Blue bars are given ranges of desired step length $[l_{min}, l_{max}]$, indicating the size of the footholds.

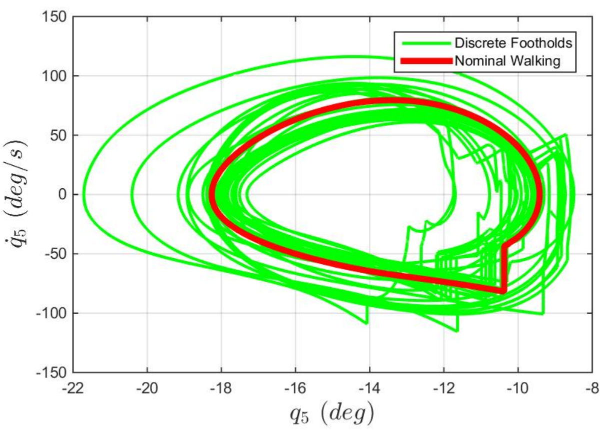


Fig. 8. Phase plot of RABBIT walking over 30 random discrete footholds. The figure illustrates torso velocity versus the torso angle. The thick red line depicts the nominal limit cycle of the periodic walking gait for comparison.

incorporating not only the mass and inertia of the links and rotors, but also friction in the various joints and transmission. Furthermore, we note that this model is not a reduced-order model like the inverted pendulum or the spring-loaded inverted pendulum (SLIP). Finally, although we are illustrating our controller particularly for RABBIT, the proposed method is generalizable and applicable to a variety of bipedal robots with various morphologies.

For RABBIT, the stance phase is parametrized by a suitable set of coordinates, given by $q := (q_1, q_2, q_3, q_4, q_5)$ as illustrated in Fig. 4. Here, q_1 and q_2 are the femur angles (referenced to the torso), q_3 and q_4 are the knee angles, and q_5 is the absolute angle of the torso. Because RABBIT has point feet (while many other legged robots have flat feet), the stance phase dynamics are underactuated with the system possessing 4 actuated degrees-of-freedom (DOF) and 1 underactuated DOF.

For the problem of avoiding overhead obstacles, the simulation result for the simpler constraint (35) and for the more involved constraint (38) is illustrated in Fig. 5.

For the problem of walking over discrete footholds, we will consider a nominal periodic walking gait or virtual constraint that will provide a stable walking gait for RABBIT. This nominal gait has a step length of $0.45m$. We will run the simulation in multiple steps and apply CBF to change the step length as detailed in Sec. 4.3. Desired values of step length will be chosen randomly in the range of $0.25m$ to $0.60m$, corresponding to a -44% to $+33\%$ change in the nominal step length of the gait. These values were empirically determined as the regions for which the controller can guarantee safety while respecting the hardware torque saturation without switching from the nominal gait to another gait. Fig. 6 illustrates three sample runs of 30 walking steps. As can be seen from this figure, the desired range $[l_{min}, l_{max}]$ is changed randomly for every step. With the proposed feedback control method, the robot's step length at impact always satisfies this randomly chosen range for each step. Fig. 7 illustrates the stick figure of RABBIT for 5 steps of walking, clearly illustrating the precise foot placement on the discrete footholds. Note that the proposed CLF-CBF-QP is solved in under $1ms$ and lends itself easily to experimental implementations at real-time speeds.

6. CONCLUSION

In summary, we have presented a novel nonlinear feedback control that provides stability guarantees through Lyapunov functions and critical safety guarantees through Barrier functions. The proposed method is illustrated on the problem of dynamic walking over a terrain of randomly generated discrete footholds that are separated by $0.25m$ to $0.6m$, corresponding to -44% to $+33\%$ change in the nominal gait's step length. The controller guarantees critical safety and respects torque saturation, while still retaining stability guarantees of dynamic walking for a nonlinear, hybrid, underactuated, five-link planar bipedal robot. The proposed feedback controller is easily applicable to more broader class of CPSs as well.

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