L_1 Adaptive Control for Bipedal Robots with Control Lyapunov Function based Quadratic Programs

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***Abstract*— This paper presents an approach to apply L_1 adaptive control for output regulation in the presence of nonlinear uncertainty in underactuated hybrid systems with application to bipedal walking. The reference model is generated by control Lyapunov function based quadratic program (CLF-QP) controller and is nonlinear. We evaluate our proposed control design on a model of RABBIT, a five-link planar bipedal robot. The result is the exponential stability of the robot with an unchanged rate of convergence under different levels of model uncertainty.**

I. INTRODUCTION

In recent years, the introduction of L_1 adaptive control technique has enabled decoupling of adaptation and robustness in adaptive control techniques. In particular, by applying a low-pass filter as part of the adaptation laws help the L_1 adaptive controller to guarantee not only stability [3] but also transient performance [5]. L_1 adaptive control appears to have great potential for application in aerospace systems, illustrated in [6], [10]. However, to the best of our knowledge, using L_1 adaptive control to deal with uncertainty for control of bipedal robots, systems that are hybrid, high-dimensional, nonlinear and undeactuated, has not been considered. Furthermore, while standard L_1 adaptive control typically solves the problem of tracking a given linear reference system, in this paper, we present an adaptive control for nonlinear uncertainty with a nonlinear reference model that arises as the closed-loop system on application of a rapidly exponential stabilizing control Lyapunov function (RES-CLF) [1]. For control of bipedal robots, guaranteeing a suitable rate of convergence is very important for the stability of its hybrid dynamics. That is the reason why we need to drive the robot to follow a fast reference dynamics. The presence of a low-pass filter in the L_1 adaptive control allows us to prevent high-frequency control signals that are typical and frequently seen in adaptive control problems. This will be critical to keep motor torques less noisy, and will contribute to ensuring the validity of the unilateral ground contact constraints, as well as retaining the energy efficiency of walking control of bipedal robot.

There have been several approaches for control of bipedal robots. The method of Hybrid Zero Dynamics (HZD), [13], [14], has been very successful in dealing with the hybrid and underactuated dynamics of legged locomotion. This method is characterized by choosing a set of output functions, which when driven to zero, creates a lower-dimensional time-invariant zero dynamics manifold. Stable periodic orbits designed on this lower-dimensional system are then also stable orbits for the full system under an appropriate controller. Until recently, experimental implementations of the HZD method relied on input-output linearization with PD control to drive the system to the zero dynamics manifold, for instance see dynamic walking [11] and running [12] on MABEL. However, recent work on control Lyapunov function (CLF)-based controllers has enabled effective implementations of stable walking, both in simulations and experiments [1]. This flexible control design, based on Lyapunov theory, has also enabled computing the control based on online quadratic programs (QPs), facilitating incorporating additional constraints into the control computation. For instance, control Lyapunov function based quadratic programs (CLF-QPs) with constraints on torque saturation were demonstrated experimentally in [7], and CLF-QPs were used to design a unified controller for performing locomotion and manipulation tasks in [2]. Sufficient conditions for Lipschitz continuity of the control produced by solving the CLF-QP problem are reported in [9].

However, all these controllers assume a perfect knowledge of the dynamic model. Furthermore, there is tremendous interest in employing legged and humanoid robots for dangerous missions in disaster and rescue scenarios. This is evidenced by the ongoing grand challenge in robotics, The DARPA Robotics Challenge (DRC). Such time and safety critical missions require the robot to operate swiftly and stably while dealing with high levels of uncertainty and large external disturbances. The limitation of current research, as well as the demand of practical requirement, motivates our research on adaptive control for hybrid systems in general and bipedal robots in particular.

The rest of the paper is organized as follows. Section II revisits rapidly exponentially stabilizing control Lyapunov functions (RES-CLFs), and control Lyapunov function-based quadratic programs (CLF-QPs). Section III discusses the adverse effects of uncertainty in the dynamics on the CLF-QP controllers. Section IV presents the proposed L_1 adaptive controller with CLF-QP. Section V develops the proposed L_1 adaptive controller with CLF-QP to cope with additional constraint of torque saturation. Section VI presents simulations of the controllers on a perturbed model of RABBIT, a five-link planar bipedal robot. Finally, Section VII provides concluding remarks.

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II. Rapidly Exponentially Stabilizing Control Lyapunov Functions and Quadratic Programs Revisited

A. Hybrid Model

Bipedal walking is characterized by single-support continuous-time dynamics and double-support discrete-time impact dynamics, and is represented by a hybrid model,

$$\mathcal{H} = \begin{cases} \dot{x} = f(x) + g(x)u, & x^- \notin S, \\ x^+ = \Delta(x^-), & x^- \in S, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the robot state and control inputs respectively, x^- and x^+ represent the state before and after impact, S represents the switching surface when the swing leg contacts the ground, and Δ represents the discrete-time impact map. We also define output functions $y(x) \in \mathbb{R}^m$, to represent the walking gait, such that the method of Hybrid Zero Dynamics (HZD) drives these output functions (and their first derivatives) to zero, thereby imposing "virtual constraints" such that the system evolves on the lower-dimensional zero dynamics manifold, given by

$$Z = \{x \in \mathbb{R}^n \mid y(x) = 0,\ L_f y(x) = 0\}. \tag{2}$$

B. Input-Output Linearization

If $y(x)$ has vector relative degree 2, then the second derivative takes the form

$$\ddot{y} = L_f^2 y(x) + L_g L_f y(x)\ u. \tag{3}$$

Suppose, $(\eta, z) = \Phi(x)$ is a state transformation, where the transverse variables $\eta = [y,\ \dot{y}]^T$, and $z \in Z$. Then, using the input-output linearizing pre-control

$$u(x) = (L_g L_f y(x))^{-1} \left(-L_f^2 y(x) + \mu\right), \tag{4}$$

we obtain the closed-loop dynamics in terms of (η, z), as

$$\begin{array}{c} \dot{\eta} = F\eta + G\mu \\ \dot{z} = p(\eta, z) \end{array}, \quad F = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ I \end{bmatrix}. \tag{5}$$

We can then apply the PD control

$$\mu = \begin{bmatrix} -\frac{1}{\varepsilon^2} K_P & -\frac{1}{\varepsilon} K_D \end{bmatrix} \eta, \tag{6}$$

which will exponentially stablilize the the system if

$$A = \begin{bmatrix} 0 & I \\ -\frac{1}{\varepsilon^2} K_P & -\frac{1}{\varepsilon} K_D \end{bmatrix}. \tag{7}$$

is Hurwitz. The ε controls the rate of convergence and is needed to counteract the expansive impact map Δ to guarantee exponential stability of the hybrid system.

C. CLF-based Control

We present a controller introduced in [1], that provides guarantees of *rapid exponential stability* for the traverse variables η. In particular, a function $V_\varepsilon(\eta)$ is a *rapidly exponentially stabilizing control Lyapunov function (RES-CLF)* for the system (5) if there exist positive constants $c_1, c_2, c_3 > 0$ such that for all $0 < \varepsilon < 1$ and all states (η, z) it holds that

$$c_1 \|\eta\|^2 \le V_\varepsilon(\eta) \le \frac{c_2}{\varepsilon^2} \|\eta\|^2, \tag{8}$$

$$\dot{V}_\varepsilon(\eta, \mu) + \frac{c_3}{\varepsilon} V_\varepsilon(\eta) \le 0. \tag{9}$$

We chose a CLF candidate as follows

$$V_\varepsilon(\eta) = \eta^T \begin{bmatrix} \frac{1}{\varepsilon} I & 0 \\ 0 & I \end{bmatrix} P \begin{bmatrix} \frac{1}{\varepsilon} I & 0 \\ 0 & I \end{bmatrix} \eta =: \eta^T P_\varepsilon \eta, \tag{10}$$

where P is the solution of the Lyapunov equation $A^T P + PA = -Q$ (where A is defined in (7) and Q is any symmetric positive-definite matrix). We then have,

$$\dot{V}_\varepsilon(\eta, \mu) + \frac{c_3}{\varepsilon} V_\varepsilon(\eta) =: \psi_{0,\varepsilon}(\eta, z) + \psi_{1,\varepsilon}(\eta, z)\mu_\varepsilon, \tag{11}$$

where,

$$\begin{aligned} \psi_{0,\varepsilon}(\eta, z) &= \eta^T (F^T P_\varepsilon + P_\varepsilon F)\eta + \frac{c_3}{\varepsilon} V_\varepsilon(\eta, z) \\ \psi_{1,\varepsilon}(\eta, z) &= 2\eta^T P_\varepsilon G. \end{aligned} \tag{12}$$

We can then construct the control μ that satisfies the RES condition (9) as follows: We define the set, $K_\varepsilon(\eta, z) = \{\mu_\varepsilon \in \mathbb{R}^m : \psi_{0,\varepsilon}(\eta, z) + \psi_{1,\varepsilon}(\eta, z)\mu_\varepsilon \le 0\}$. Then, it can be show that for any Lipschitz continuous feedback control law $\mu \in K_\varepsilon(\eta, z)$ (min-norm or Sontag control [1]), it holds that

$$\|\eta(t)\| \le \frac{1}{\varepsilon} \sqrt{\frac{c_2}{c_1}} e^{-\frac{c_3}{2\varepsilon} t} \|\eta(0)\|, \tag{13}$$

i.e. the rate of exponential convergence to the zero dynamics manifold can be directly controlled with the constant ε through $\frac{c_3}{\varepsilon}$.

D. CLF-based Quadratic Programs

CLF-based quadratic programs (QPs) were introduced in [7], where it was identified that a controller $\mu \in K_\varepsilon(\eta, z)$ can be directly selected through an online QP to satisfy (9):

CLF-QP:

$$\begin{aligned} \underset{\mu}{\operatorname{argmin}} \quad & \mu^T \mu \\ \text{s.t.} \quad & \psi_{0,\varepsilon}(\eta, z) + \psi_{1,\varepsilon}(\eta, z)\ \mu \le 0. \end{aligned} \tag{14}$$

Furthermore, the QP formulation also enables incorporating additional constraints, such as strict torque saturation constraints.

Having presented recent developments in control Lyapunov functions and control Lyapunov functions based quadratic programs for hybrid dynamical systems, we next consider the effect of uncertainty in the dynamics on these controllers.

III. Adverse Effects of Uncertainty in Dynamics on the CLF-QP controller

The CLF-based approaches presented in Section II have several interesting properties. Firstly, they provide a guarantee on the exponential stability of the hybrid system, they are optimal with respect to some cost function, result in the minimum control effort, and provide means of balancing conflicting requirements between performance and state-based constraints. These controllers were even successfully implemented on MABEL, see [1], [7]. However, a primary disadvantage of these controllers is that they require an accurate dynamical model of the system. Specifically, as we will see, even for a simpler bipedal model such as RABBIT (compared to MABEL), uncertainty in mass and inertia properties of the model can cause bad control quality leading to tracking errors, and could potentially lead to walking that is unstable.

If we consider uncertainty in the dynamics and assume that the functions, $f(x), g(x)$ of the real dynamics (1), are unknown, we then have to design our controller based on nominal functions $\tilde{f}(x), \tilde{g}(x)$. Thus, the pre-control law (4) is reformulated as

$$u(x) = (L_{\tilde{g}} L_{\tilde{f}} y(x))^{-1} \left(-L_{\tilde{f}}^2 y(x) + \mu \right). \tag{15}$$

Substituting $u(x)$ from (15) into (3), the second derivative of the output, $y(x)$, then becomes

$$\ddot{y} = \mu + \theta, \tag{16}$$

where,

$$\begin{aligned} \theta &= \Delta_1 + \Delta_2 \mu \\ \Delta_1 &= L_f^2 y(x) - L_g L_f y(x) (L_{\tilde{g}} L_{\tilde{f}} y(x))^{-1} L_{\tilde{f}}^2 y(x), \\ \Delta_2 &= L_g L_f y(x) (L_{\tilde{g}} L_{\tilde{f}} y(x))^{-1} - I. \end{aligned} \tag{17}$$

The closed-loop system now takes the form

$$\dot{\eta} = F\eta + G(\mu + \theta). \tag{18}$$

where F and G are defined in (5).

Clearly for $\theta \neq 0$, the closed-loop system does not have an equilibrium and therefore the PD control (6) does not stabilize the system dynamics. This raises the question of whether it's possible for controllers to account for this model uncertainty, and if so, how do we design such a controller. Note that the uncertainty θ is a nonlinear function of (x, μ), and therefore a nonlinear function of η and time (since $(\eta, z) = \Phi(x), \mu = \mu(\eta), z = z(t)$).

IV. L1 Adaptive Control with Control Lyapunov Function based Quadratic Program

From the Section III, we have the system with uncertainty described by (18) where the nonlinear uncertainty $\theta = \theta(\eta, t)$. As a result, for every time t, we can always find out $\alpha(t)$ and $\beta(t)$ such that [4]:

$$\theta(\eta, t) = \alpha(t) ||\eta|| + \beta(t) \tag{19}$$

The principle of our method is to design a combined controller $\mu = \mu_1 + \mu_2$, where μ_1 is to control the model to follow the desired reference model and μ_2 is to compensate the nonlinear uncertainty θ. The reference model could be linear when we apply conventional PD control (6) for μ_1.

In the perfect case of without uncertainty, we will have the following desired linear model with PD control

$$\dot{\eta} = A_m \eta \tag{20}$$

where $A_m = A$ in (7), which is a standard linear reference model for L_1 adaptive control.

In this paper, we present a method to consider a reference model for L_1 adaptive control that arises from a rapidly exponentially stabilizing CLF-based controller. In particular, we consider the reference model that arises when μ_1 is chosen to be the solution of the QP (14). This reference model is nonlinear and has no closed-form analytical expression.

The state predictor can then be expressed as follows,

$$\dot{\hat{\eta}} = F\hat{\eta} + G\hat{\mu}_1 + G(\mu_2 + \hat{\theta}), \tag{21}$$

where,

$$\hat{\theta} = \hat{\alpha} ||\eta|| + \hat{\beta}, \tag{22}$$

and $\hat{\mu}_1$ is computed as the solution of the following QP:

CLF-QP for the state predictor:

$$\begin{aligned} \underset{\hat{\mu}_1, d_1}{\operatorname{argmin}} \quad & \hat{\mu}_1^T \hat{\mu}_1 \\ \text{s.t.} \quad & \psi_{0,\varepsilon}(\hat{\eta}, z) + \psi_{1,\varepsilon}(\hat{\eta}, z)\ \hat{\mu}_1 \leq 0. \end{aligned} \tag{23}$$

In order to compensate the estimated uncertainty $\hat{\theta}$, we can just simply choose $\mu_2 = -\hat{\theta}$ to obtain

$$\dot{\hat{\eta}} = F\hat{\eta} + G\hat{\mu}_1 \tag{24}$$

which satisfies the rapid exponential stability [1] since $\hat{\mu}_1$ is designed through the CLF-QP in (23). However, $\hat{\theta}$ typically has high-frequency content due to fast estimation. For the reliability of the control scheme and in particular to not violate the unilateral ground force constraints for bipedal walking, it is very important to not have high-frequency content in the control signals. Thus, we apply the L_1 adaptive control scheme to decouple estimation and adaptation [5]. Therefore, we will have

$$\mu_2 = -C(s)\hat{\theta} \tag{25}$$

where $C(s)$ is a low-pass filter with magnitude being 1.

Define the difference between the real model and the reference model $\tilde{\eta} = \hat{\eta} - \eta$, we then have,

$$\dot{\tilde{\eta}} = F\tilde{\eta} + G\tilde{\mu}_1 + G(\tilde{\alpha}||\eta|| + \tilde{\beta}), \tag{26}$$

where

$$\tilde{\mu}_1 = \hat{\mu}_1 - \mu_1,\ \tilde{\alpha} = \hat{\alpha} - \alpha,\ \tilde{\beta} = \hat{\beta} - \beta. \tag{27}$$

As a result, we will estimate θ indirectly through α and β, or the values of $\hat{\alpha}$ and $\hat{\beta}$ computed by the following adaptation laws based on the projection operators [8],

$$\begin{aligned}\dot{\hat{\alpha}} &= \Gamma\mathbf{Proj}(\hat{\alpha}, y_\alpha),\\ \dot{\hat{\beta}} &= \Gamma\mathbf{Proj}(\hat{\beta}, y_\beta).\end{aligned} \tag{28}$$

where Γ is a symmetric positive define matrix.

We now have the control diagram of the L_1 adaptive control with CLF-QP described in Fig. 1.

In order to find out a suitable function y_α and y_β for the adaptation laws in (28), we will consider the following control Lyapunov candidate function

$$\tilde{V} = \tilde{\eta}^T P_\varepsilon \tilde{\eta} + \tilde{\alpha}^T \Gamma^{-1} \tilde{\alpha} + \tilde{\beta}^T \Gamma^{-1} \tilde{\beta} \tag{29}$$

Because $\tilde{\eta} = \hat{\eta} - \eta$ satisfies the RES condition imposed by the two CLF-QP (14) and (23), it implies that

$$(F\tilde{\eta} + G\tilde{\mu}_1)^T P_\varepsilon \tilde{\eta} + \tilde{\eta}^T P_\varepsilon (F\tilde{\eta} + G\tilde{\mu}_1) \leq -\frac{c_3}{\varepsilon}\tilde{\eta}^T P_\varepsilon \tilde{\eta} \tag{30}$$

Furthermore, with the property of projection operator [8], we have:

$$\begin{aligned}(\hat{\alpha} - \alpha)^T(\mathbf{Proj}(\hat{\alpha}, y_\alpha) - y_\alpha) &\leq 0,\\ (\hat{\beta} - \beta)^T(\mathbf{Proj}(\hat{\beta}, y_\beta) - y_\beta) &\leq 0.\end{aligned} \tag{31}$$

So, if we choose the projection functions y_α and y_β as,

$$\begin{aligned}y_\alpha &= -GP_\varepsilon \tilde{\eta}||\eta||,\\ y_\beta &= -GP_\varepsilon \tilde{\eta},\end{aligned} \tag{32}$$

then from (30), (31), we will have

$$\begin{aligned}\dot{\tilde{V}} + \frac{c_3}{\varepsilon}\tilde{V} \leq \frac{c_3}{\varepsilon}\tilde{\alpha}^T\Gamma^{-1}\tilde{\alpha} + \frac{c_3}{\varepsilon}\tilde{\beta}^T\Gamma^{-1}\tilde{\beta}\\ -\tilde{\alpha}^T\Gamma^{-1}\dot{\alpha} - \dot{\alpha}^T\Gamma^{-1}\tilde{\alpha}\\ -\tilde{\beta}^T\Gamma^{-1}\dot{\beta} - \dot{\beta}^T\Gamma^{-1}\tilde{\beta}.\end{aligned} \tag{33}$$

We assume that the uncertainties α, β and their time derivatives are bounded. Furthermore, the projection operators (28) will also keep $\tilde{\alpha}$ and $\tilde{\beta}$ bounded (see [4] for a detailed proof about these properties.) We define these bounds as follows:

$$\begin{aligned}||\tilde{\alpha}|| \leq& \tilde{\alpha}_b, \quad ||\tilde{\beta}|| \leq \tilde{\beta}_b,\\ ||\dot{\alpha}|| \leq& \dot{\alpha}_b, \quad ||\dot{\beta}|| \leq \dot{\beta}_b.\end{aligned} \tag{34}$$

Combining this with (33), we have,

$$\dot{\tilde{V}} + \frac{c_3}{\varepsilon}\tilde{V} \leq \frac{c_3}{\varepsilon}\delta_{\tilde{V}}, \tag{35}$$

where

$$\delta_{\tilde{V}} = 2||\Gamma||^{-1}(\tilde{\alpha}_b^2 + \tilde{\beta}_b^2 + \frac{\varepsilon}{c_3}\tilde{\alpha}_b\dot{\alpha}_b + \frac{\varepsilon}{c_3}\tilde{\beta}_b\dot{\beta}_b). \tag{36}$$

Thus, if $\tilde{V} \geq \delta_{\tilde{V}}$ then $\dot{\tilde{V}} \leq 0$. As a result, we always have $\tilde{V} \leq \delta_{\tilde{V}}$. In other words, by choosing the adaptation gain Γ sufficiently large, we can limit the Control Lyapunov Function (29) in an arbitrarily small neighborhood $\delta_{\tilde{V}}$ of the origin. Therefore the tracking errors between the dynamics

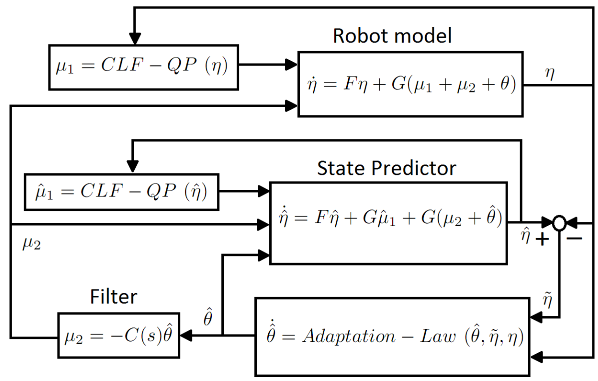


Fig. 1: Control diagram illustrating L_1 adaptive control with a CLF-QP based closed-loop reference model.

model (18) and the reference model (21), $\tilde{\eta}$, and the error between the real and estimated uncertainty, $\tilde{\alpha}$, $\tilde{\beta}$ are bounded as follows:

$$||\tilde{\eta}|| \leq \sqrt{\frac{\delta_{\tilde{V}}}{||P_\varepsilon||}}, ||\tilde{\alpha}|| \leq \sqrt{||\Gamma||\delta_{\tilde{V}}}, ||\tilde{\beta}|| \leq \sqrt{||\Gamma||\delta_{\tilde{V}}}. \tag{37}$$

Another interesting property of this controller is that $\delta_{\tilde{V}}$ can also be decreased by choosing a sufficiently small $\varepsilon < \bar{\varepsilon}$.

V. L_1 Adaptive Control with Control Lyapunov Function based Quadratic Program and Torque Saturation

CLF-QP based controllers can be extended to incorporate other constraints, such as strict torque constraints as carried out in [7]. This can also be combined with L_1 adaptive control. The controller design is almost equivalent to the L_1 adaptive control with CLF-QP presented in Section IV. We retain μ_2 as in (25) and adaptation laws for $\hat{\alpha}, \hat{\beta}$ as in (28), while we redesign μ_1 and $\hat{\mu}_1$ based on the CLF-QP with torque saturation [7], as below:

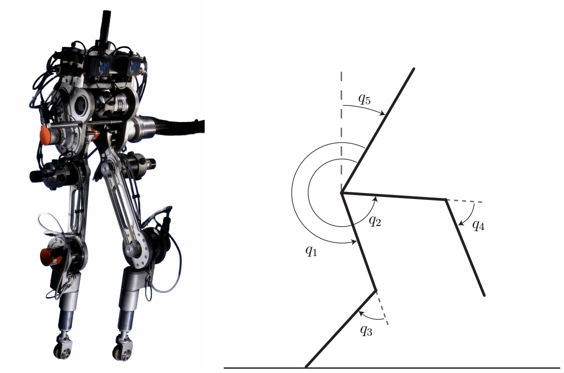
CLF-QP with torque saturation for the dynamics model:

$$\begin{aligned}\underset{\mu_1, d_1}{\text{argmin}} \quad & \mu_1^T\mu_1 + p\lambda^2\\ \text{s.t.} \quad & \psi_{0,\varepsilon}(\eta, z) + \psi_{1,\varepsilon}(\eta, z)\ \mu_1 \leq \lambda,\\ & (L_g L_f y(q,\dot{q}))^{-1}\ \mu_1 \geq (u_{min} - u^*),\\ & (L_g L_f y(q,\dot{q}))^{-1}\ \mu_1 \leq (u_{max} - u^*).\end{aligned} \tag{38}$$

CLF-QP with torque saturation for the state predictor:

$$\begin{aligned}\underset{\hat{\mu}_1, d_1}{\text{argmin}} \quad & \hat{\mu}_1^T\hat{\mu}_1 + p\lambda^2\\ \text{s.t.} \quad & \psi_{0,\varepsilon}(\hat{\eta}, z) + \psi_{1,\varepsilon}(\hat{\eta}, z)\ \hat{\mu}_1 \leq \lambda,\\ & (L_g L_f y(q,\dot{q}))^{-1}\ \hat{\mu}_1 \geq (u_{min} - u^*),\\ & (L_g L_f y(q,\dot{q}))^{-1}\ \hat{\mu}_1 \leq (u_{max} - u^*).\end{aligned} \tag{39}$$

Here, λ is a relaxation of the stability criterion to respect potentially conflicting torque saturation constraints, and p the penalty for relaxation. Note, that the proposed controller only respects saturation for the CLF-QP controller component (μ_1), and not the L1 adaptive controller component (μ_2).



(a) RABBIT (b) Coordinate system

Fig. 2: (a) RABBIT, a planar five-link bipedal robot with nonlinear, hybrid and underactuated dynamics. (b) The the associated generalized coordinate system used, where q_1, q_2 are the relative stance and swing leg femur angles referenced to the torso, q_3, q_4 are the relative stance and swing leg knee angles, and q_5 is the absolute torso angle in the world frame.

VI. Simulation

Having developed the L_1 adaptive control with CLF-QP, both with and without torque saturation (see Sections IV, V), we now demonstrate the performance of these controllers through numerical simulations and offer comparisions with the standard CLF-QP controller in [1], [7]. We will conduct simulations with the following controllers:

$$\begin{aligned} &Controller\ A: \text{CLF-QP} \\ &Controller\ B: L_1\text{-CLF-QP} \\ &Controller\ C: L_1\text{-CLF-QP with torque saturation} \end{aligned} \tag{40}$$

For Controller 'C', we choose the following torque saturations: $u_{max} = u_b; u_{min} = -u_b$ with $u_b = \begin{bmatrix} 65 & 65 & 65 & 65 \end{bmatrix}^T$.

We perform simulations using a model of RABBIT, wherein, the stance phase is parametrized by a suitable set of coordinates as illustrated in Fig. 2. Here, q_1 and q_2 are the thigh angles (referenced to the torso), q_3 and q_4 are the knee angles, and q_5 is the absolute angle of the torso. Because RABBIT has point feet, the stance phase dynamics are underactuated with the system possessing 4 actuated degrees-of-freedom (DOF) and 1 underactuated DOF.

For the purpose of evaluating the L_1 adaptive control with CLF-QP controller, we will consider a periodic walking gait and an associated controller that is developed for a nominal model of RABBIT.The simulation is then carried out on a perturbed model of RABBIT, where the perturbation is introduced by scaling all mass and inertia parameters of each link by a fixed constant scale factor. The perturbed model is unknown to the controller and will serve as an uncertainty injected into the model. We will illustrate three separate cases of scaling the mass and inertia:

$$\begin{cases} Case\ I\ \ : & \text{model scale = 1} \\ Case\ II\ : & \text{model scale = 0.7} \\ Case\ III: & \text{model scale = 1.5} \end{cases} \tag{41}$$

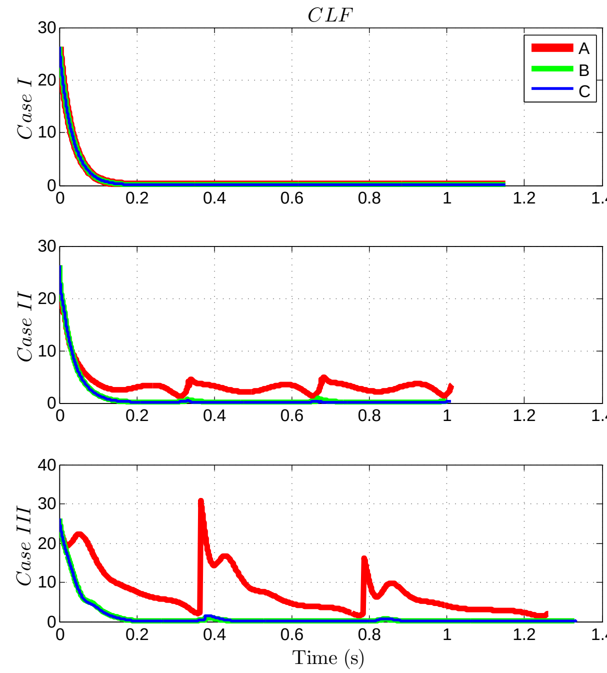


Fig. 3: Control Lyapunov function for the three controllers A-C indicated in (40), for the cases I-III (41) of model perturbations. The simulation is for three walking steps.

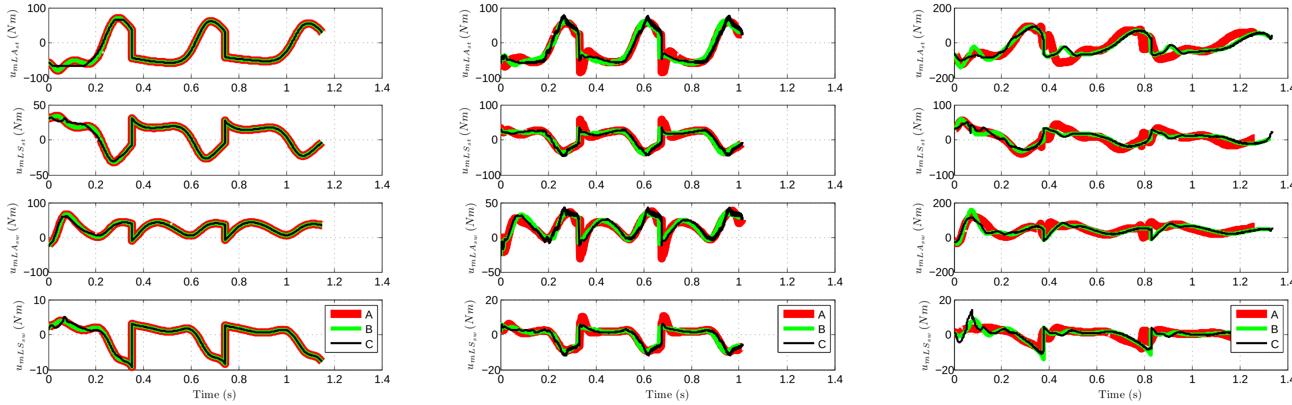
As we can see from the Fig. 3, in Case I, when we set the model scale equal to 1, i.e., no uncertainty, the performance of the three controllers in (40) are nearly the same. However, from Fig. 4, we can notice that the control inputs of the controller C are limited by the torque saturation u_b, resulting in a bit slower rate of convergence of tracking errors.

When we present a high level of uncertainty, Cases II-III with model scale = 0.7 and 1.5, Controller 'A' cannot guarantee a zero tracking error. However, Controllers 'B' and 'C', not only drive the output y to converge to zero but also keep the rate of convergence unchanged through the three cases of model uncertainty. This property is important for bipedal walking since a sufficiently fast rate of convergence is required to guarantee stability of the hybrid system [1]. The rates of convergence of controller 'C' in the Cases II-III are a bit slower than those of the controller 'B' due to the additional constraint of torque saturation.

We note that, the performance of the two L_1 adaptive controllers ('B' and 'C'), are much better than the standard CLF-QP controller ('A'). Further, although the control signals of the three controller are similar, as is evident from close observation of Fig. 4, Controller 'C', the torque is saturated. Finally, Fig. 5 illustrates the phase plot of the torso angle for the three controllers with a model uncertainty corresponding to case III.

VII. Conclusion

In summary, we have presented a novel control methodology to apply L_1 adaptive control for dynamic bipedal walking in the presence of uncertainty. The controller explicitly



(a) Case I: model scale = 1 (b) Case II: model scale = 0.7 (c) Case III: model scale = 1.5

Fig. 4: Control inputs (motor torques for stance and swing legs) based on the simulation of three cases of perturbed model of RABBIT (41) with three controllers as described in (40). Simulation of three walking steps are shown.

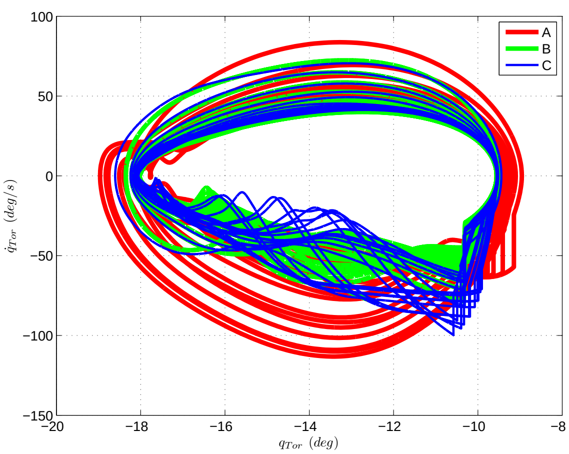


Fig. 5: Phase portrait of the torso angle for walking simulation for 20 steps for model perturbation Case III (model scale = 1.5) with three different controllers (A-C). Note that the uncertainty causes a change in the periodic orbit. This is as expected, as the controller only tracks the outputs (even in the presence of uncertainty), and the unactuated dynamics on Z evolve passively.

considers the nonlinear, underactuated and hybrid dynamics that are characteristic of bipedal robots. The proposed control strategy uses a control Lyapunov function based controller to create a closed-loop nonlinear reference model for the L_1 adaptive controller for working in the presence of uncertainty. Numerical simulations on RABBIT demonstrate the validity of the proposed controller.

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