Control Lyapunov Function based Quadratic Programs for Torque Saturated Bipedal Walking

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***Abstract*—This work builds off of recent work on rapidly exponentially stabilizing control Lyapunov functions (RES-CLF) and presents a novel method to address actuator saturation in bipedal walking. The proposed method employs quadratic programming (QP) to implement CLF-based controllers, enabling directly incorporating user-specified input bounds into the controller. We show that even with increasingly stringent levels of actuator saturation, there is only a gradual degradation of performance while still maintaining the task of walking.**

Keywords-dynamic legged locomotion; actuator saturation; control Lyapunov functions.

I. Introduction

The method of Hybrid Zero Dynamics (HZD) [3] has been very successful in dealing with the hybrid and underactuated dynamics of legged locomotion and provides a framework for implementing formally provable controllers for achieving dynamic walking and running. Until recently, this has required either controllers with finite-time convergence or input-output linearization based controllers. Recent work on control Lyapunov function (CLF)-based controllers has enabled implementing hybrid zero dynamics using a variant of CLF, called the rapidly exponentially stabilizing control Lyapunov function (RES-CLF) [1]. This type of CLF incorporates an additional tuning parameter, enabling the control designer to directly control the rate of exponential convergence. This key feature enabled proving local exponential stability of the hybrid periodic orbit corresponding to a walking or running gait. Implementation of this controller on the bipedal robot MABEL was successful, however, user-defined actuator bounds (specified for the sake of safety) were active throughout a large portion of the walking gait. Instead of "blindly" applying such hard torque limits on the computed CLF control, this work explicitly considers the actuator saturation as part of the online control computation.

II. Approach: Control Lyapunov Function based Quadratic Programs

We consider the dynamical model of a bipedal robot and use the method of Hybrid Zero Dynamics (HZD) to design a set of output functions (called virtual constraints), such that when the outputs (and their first derivatives) are driven to zero, the system dynamics is constrained to evolve on a lower dimensional manifold.

Using such a HZD pre-control, we arrive at the closed-loop output dynamics. A control Lyapunov function is then constructed for the output dynamics such that the time-derivative of the Lyapunov function satisfies an rapidly exponential stabilizing property, i.e., $\frac{d}{dt}V \leq -\gamma/\epsilon\ V$, where γ is some positive constant, and ϵ is a design parameter to control the rate of exponential convergence. Our novel approach is in implementing a controller as a quadratic program with the above expression as an inequality constraint. These are control Lyapunov function based quadratic programs (CLF-QP), introduced in [2]. Additional constraints, such as torque saturation, can then be easily incorporated into the quadratic program.

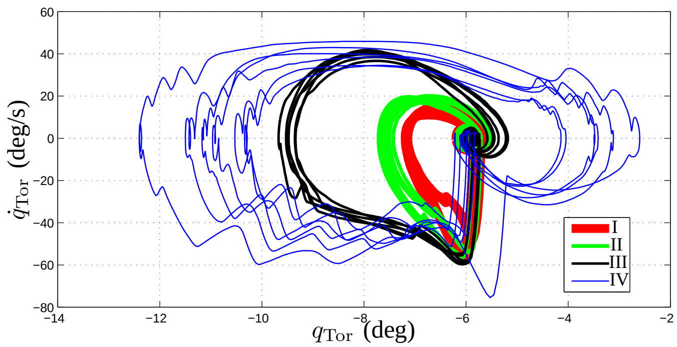

Fig. 1. Phase portrait of the torso angle for four different cases of increasing levels of input bounds. Observe that stricter saturations (increasing from case I to case IV) result in (gradual) deterioration in tracking, as evidenced by deviations of the limit cycle from the nominal orbit. However, the controller still

III. Results

CLF-QPs have been employed to successfully demonstrate stable walking both in simulations and experiments. Figure 1 considers several different bounds on the actuators, with the bounds getting more stringent from case I to case IV. In each case, simulations on MABEL demonstrate stable walking, although there is a gradual degradation in performance, as can be seen in deviation from the nominal fixed point orbit.

References

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