Dynamical walking with two and more passive degrees of freedom

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Planning and controlling a motion of a mechanical system subject to dynamic constraints is the challenging task. There are many conceptual examples in the subject, where none of available analytical methods can be applied and provide solutions. Dynamical locomotion represents one of such conceptual cases where even the task of planning a gait for a system experiencing by scenario a phase with underactuation in several degrees of freedom is difficult and commonly approached by the blind search. Similar open questions are present in planning behaviors of a robot when nominal behavior should be consistent with a certain force profile specified between some of degrees of freedom or the robot and environment.

The talk is aimed at a brief discussion of several examples with original arguments proposed in searching and efficient representing feasible motions of mechanical systems including

- two degrees of freedom passive compass-gait biped;
- three degrees of freedom compass-gait biped with a suspended by springs torso and one actuator, see Fig. 1

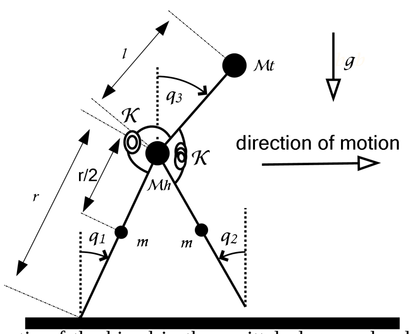

Fig. 1. Schematic of the biped in the sagittal plane on level ground. The walking motion is described by evolution of the support-leg angle q_1, the swing leg angle q_2 and the torso angle q_3. The length of each leg is denoted by r and the torso's by l. The masses of the legs, denoted by m, are lumped at $r/2$. The hip mass is denoted by M_h and the torso's by M_t, and it is lumped at a distance l from the hip. Each of the two torsional springs between the legs and the torso has stiffness coefficient K. The system is equipped with one actuator that can apply external control torque between the legs.

Both examples are important for the analysis and rich for new discoveries. Indeed, even though the compass-gait biped is one of well studied walkers in the subject several fundamental questions are left open:

- How to show that there are only two symmetric gaits for the classical settings? How to find all gaits?
- How to approximate a region of attraction for a stable gait?
- What is the mechanism of asymptotic orbital stability? To what extent the arguments of hybrid zero-dynamics, or Lyapunov function reasoning are applied? How to find such Lyapunov function for a stable gait?
- How to characterize a sensitivity of the stable gait to structural perturbations?

These and other questions become critical for the next example depicted on Fig. 1. It is likely that the system has a plenty of induced gaits and, presumably any gait of such machine should heavily exploit the passive dynamics. But how to find them? How to define a structure of a feedback controller to achieve orbital stabilization similar to observed for the passive gait of the previous example? We will partly answer some of the questions and indicate some of old and new mathematical tools helpful in solving the tasks. The following references can compliment the overview

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