

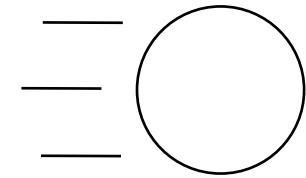
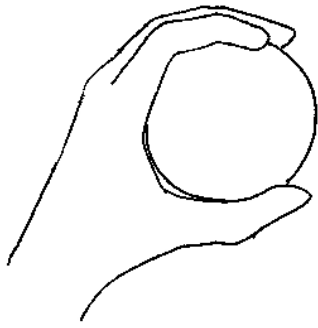
# Locomotion and Manipulation as Uncertain Hybrid Mechanical Systems

NSF Workshop on Locomotion and Manipulation: Why the Great Divide?  
April 3, 2015

Kevin M. Lynch  
Department of Mechanical Engineering  
Northwestern University



# nonprehensile manipulation



hand controls ball:  
*prehensile*

shared control:  
*nonprehensile*

environment  
controls ball

form or force  
closure grasping



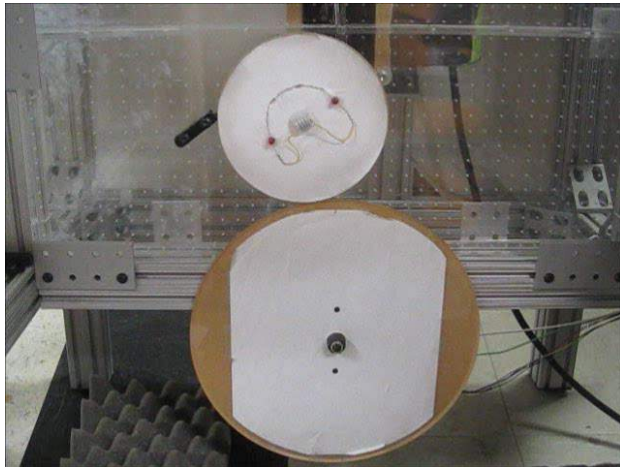
throwing and batting  
(U Tokyo)



bat juggling



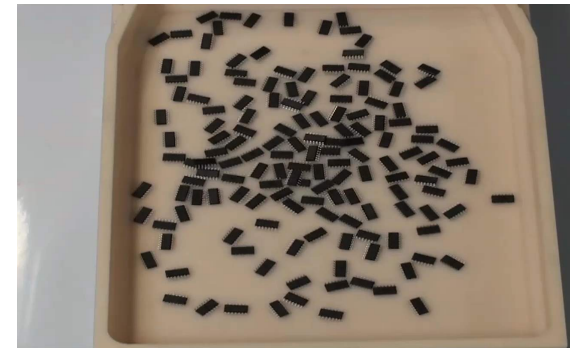
catching and palming



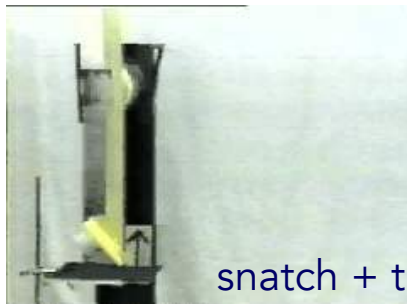
rolling and balancing



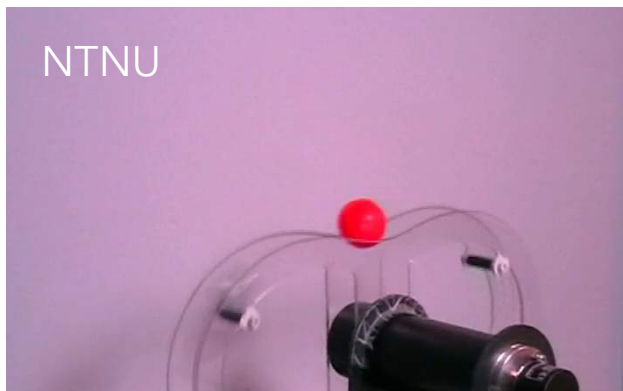
rolling (Michael Moschen)

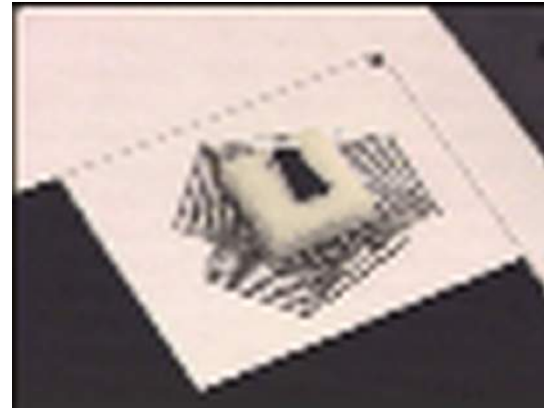
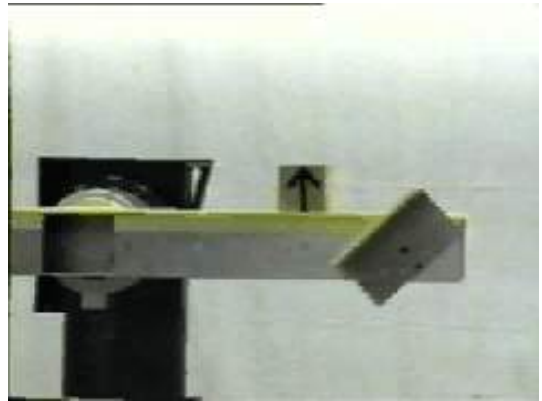
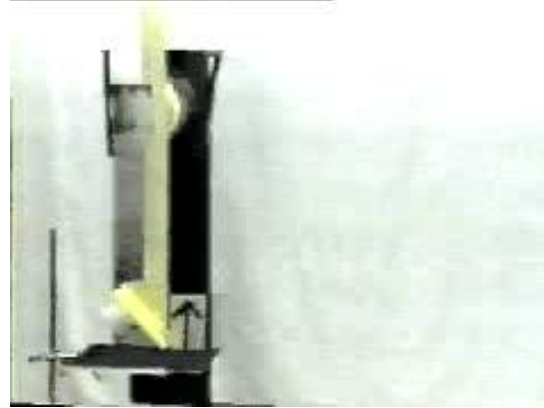
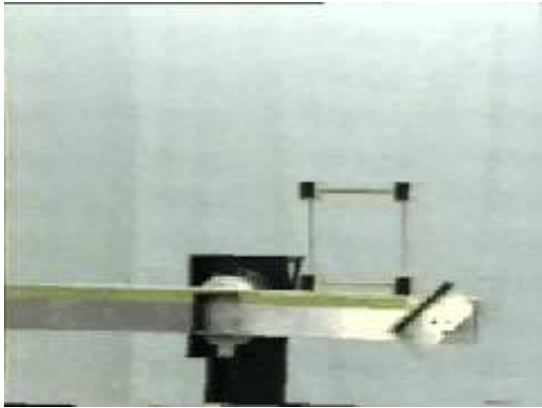


vibratory feeding (Asyri)

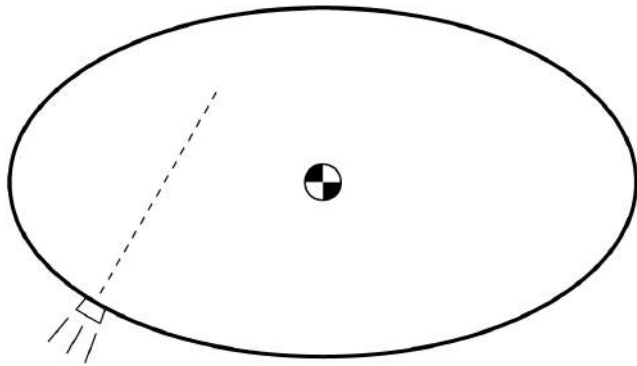


snatch + throw

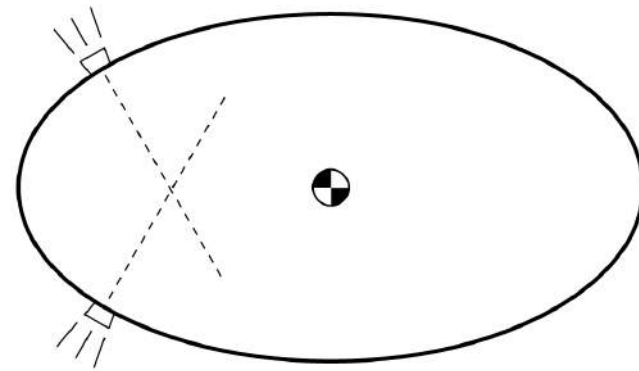




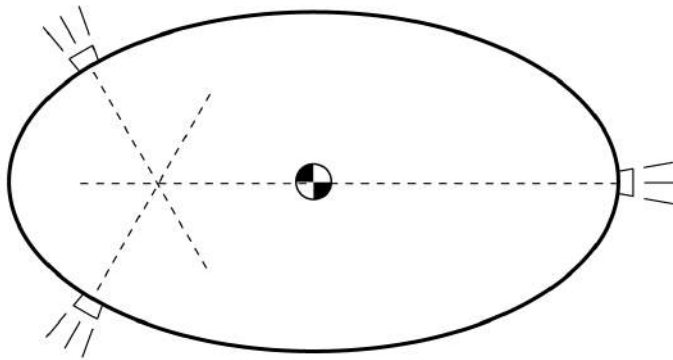
simple robots  
simple models  
rich dynamics



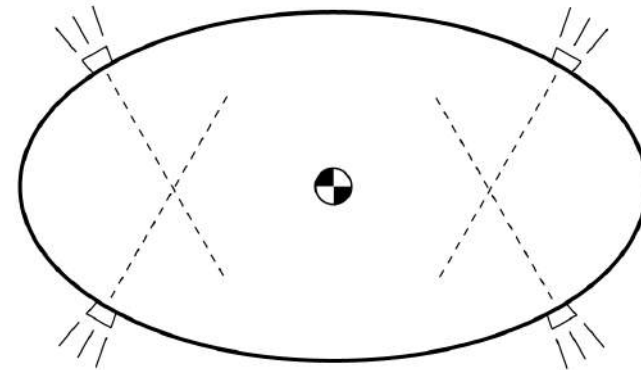
locally accessible



(globally) controllable



small-time locally controllable



linearly controllable  
(form, force closure)

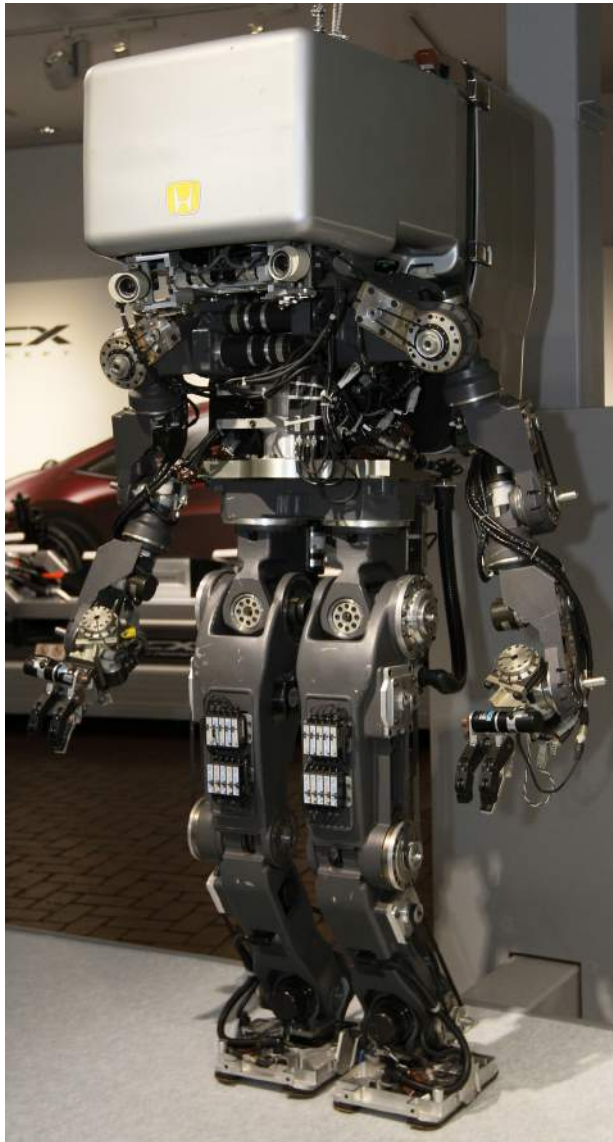
nonholo mumbo jumbo

Bullo and Lewis, *Geometric control of mechanical systems*, 2005.

Bloch, *Nonholonomic mechanics and control*, 2003.

Lynch, Bloch, Drakunov, Reyhanoglu, Zenkov. *Control of nonholonomic and underactuated systems*, 2010.

# Honda P1, 1996



## Tad McGeer

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### Abstract

*There exists a class of two-legged machines for which walking is a natural dynamic mode. Once started on a shallow slope, a machine of this class will settle into a steady gait quite comparable to human walking, without active control or energy input. Interpretation and analysis of the physics are straightforward; the walking cycle, its stability, and its sensitivity to parameter variations are easily calculated. Experiments with a test machine verify that the passive walking effect can be readily exploited in practice. The dynamics are most clearly demonstrated by a machine powered only by gravity, but they can be combined easily with active energy input to produce efficient and dextrous walking over a broad range of terrain.*

### 1. Static vs. Dynamic Walking

Research on legged locomotion is motivated partly by fundamental curiosity about its mechanics, and partly by the practical utility of machines capable of traversing uneven surfaces. Increasing general interest in robotics over recent years has coincided with the appearance of a wide variety of legged machines. A brief classification will indicate where our own work fits in. First one should distinguish between *static* and *dynamic* machines. The former maintain static equilibrium throughout their motion. This requires at least four legs and, more commonly, six. It also imposes a speed restriction, since cyclic accelerations must be limited in order to minimize inertial effects. Outstanding examples of static walkers are the Odex series (Russell 1983) and the Adaptive Suspension Vehicle (Waldron 1986). Dynamic machines, on the other hand, are more like people; they can have fewer legs than static machines, and are potentially faster.

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Vol. 9, No. 2, April 1990,  
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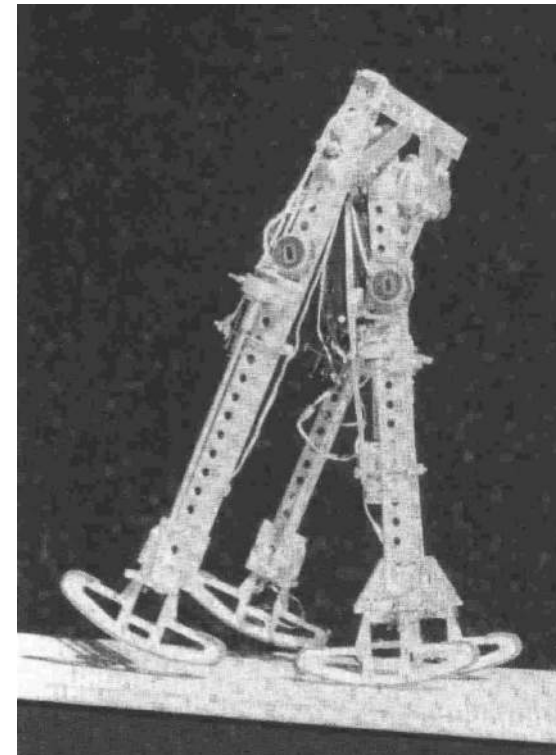
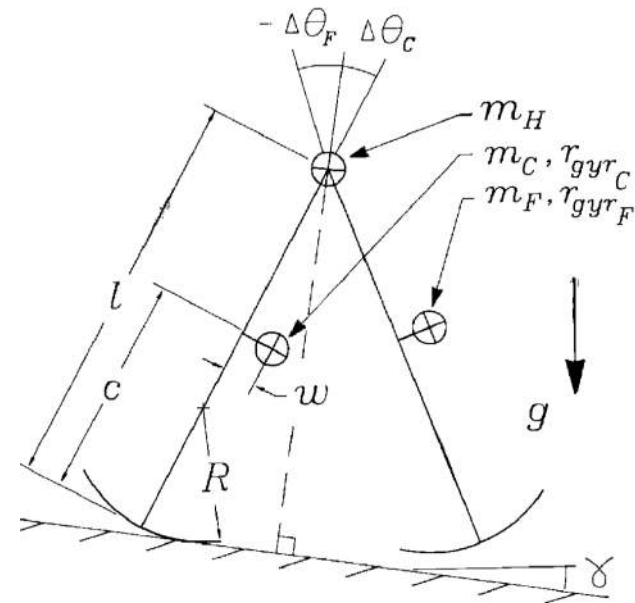
## Passive Dynamic Walking

### 2. Dynamics vs. Control

Our interest is in dynamic walking machines, which for our purposes can be classified according to the role of active control in generating the gait. At one end of the spectrum is the biped of Mita et al. (1984), whose motion is generated entirely by linear feedback control. At the end of one step, joint angles are commanded corresponding to the end of the next step, and the controller attempts to null the errors. There is no explicit specification of the trajectory between these end conditions. Yamada, Furusho, and Sano (1985) took an approach that also relies on feedback, but in their machine it is used to track a fully specified trajectory rather than just to close the gap between start and end positions. Meanwhile the stance leg is left free to rotate as an inverted pendulum, which, as we shall discuss, is a key element of passive walking. Similar techniques are used in biped walkers by Takanishi et al. (1985), Lee and Liao (1988), and Zheng, Shen, and Sias (1988).

By contrast the bipeds of Miura and Shimoyama (1984) generate their gait by feedforward rather than feedback; joint torque schedules are precalculated and played back on command. Again the stance leg is left free. However, the "feedforward" gait is unstable, so small feedback corrections are added to maintain the walking cycle. Most significantly, these are *not* applied continuously (i.e., for tracking of the nominal trajectory). Instead the "feedforward" step is treated as a processor whose output (the end-of-step state) varies with the input (the start-of-step state). Thus the feedback controller responds to an error in tracking by modifying initial conditions for subsequent steps, and so over several steps the error is eliminated. In this paper you will see analysis of a similar process. Raibert (1986) has developed comparable concepts but with a more pure implementation, and applied them with great success to running machines having from one to four legs.

All of these machines use active control in some form to generate the locomotion pattern. They can be



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## Benoit Thuilot\*

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## Bernard Espiau

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38330 Montbonnot Saint Martin, France

## A Study of the Passive Gait of a Compass-Like Biped Robot: Symmetry and Chaos

### The Simplest Walking Model: Stability, Complexity, and Scaling

Mariano Garcia

Anindya Chatterjee

Andy Ruina

Michael Coleman

Department of Theoretical and Applied Mechanics,  
212 Kimball Hall, Cornell University,  
Ithaca, NY 14853

*We demonstrate that an irreducibly simple, uncontrolled, two-dimensional, two-link model, vaguely resembling human legs, can walk down a shallow slope, powered only by gravity. This model is the simplest special case of the passive-dynamic models pioneered by McGeer (1990a). It has two rigid massless legs hinged at the hip, a point-mass at the hip, and infinitesimal point-masses at the feet. The feet have plastic (no-slip, no-bounce) collisions with the slope surface, except during forward swinging, when geometric interference (foot scuffing) is ignored. After nondimensionalizing the governing equations, the model has only one free parameter, the ramp slope  $\gamma$ . This model shows stable walking modes similar to more elaborate models, but allows some use of analytic methods to study its dynamics. The analytic calculations find initial conditions and stability estimates for period-one gait limit cycles. The model exhibits two period-one gait cycles, one of which is stable when  $0 < \gamma < 0.015$  rad. With increasing  $\gamma$ , stable cycles of higher periods appear, and the walking-like motions apparently become chaotic through a sequence of period doublings. Scaling laws for the model predict that walking speed is proportional to stance angle, stance angle is proportional to  $\gamma^{1/3}$ , and that the gravitational power used is proportional to  $v^3$  where  $v$  is the velocity along the slope.*

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JEB1788

2609

### A POINT-MASS MODEL OF GIBBON LOCOMOTION

JOHN E. A. BERTRAM<sup>1,\*</sup>, ANDY RUIA<sup>2</sup>, C. E. CANNON<sup>3</sup>, YOUNG HUI CHANG<sup>4</sup>  
AND MICHAEL J. COLEMAN<sup>5</sup>

<sup>1</sup>College of Veterinary Medicine, Cornell University, USA, <sup>2</sup>Theoretical and Applied Mechanics, Cornell University, USA, <sup>3</sup>Sibley School of Mechanical and Aerospace Engineering, Cornell University, USA, <sup>4</sup>Department of Integrative Biology, University of California-Berkeley, USA and <sup>5</sup>Sibley School of Mechanical and Aerospace Engineering, Cornell University, USA

\*Author for correspondence at Department of Nutrition, Food and Exercise Sciences, 436 Sandels Building, Florida State University, Tallahassee, FL 32306, USA (e-mail: jbertram@garnet.acns.fsu.edu)

Accepted 10 June; published on WWW 13 September 1999

## Swing and Locomotion Control for a Two-Link Brachiation Robot

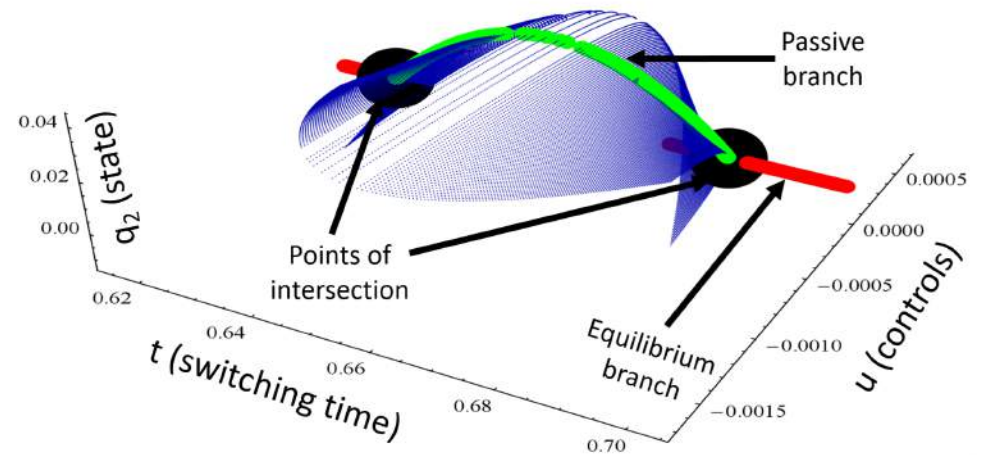
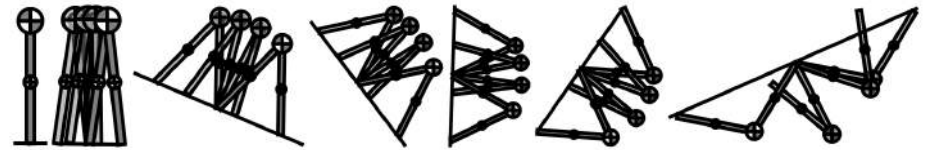
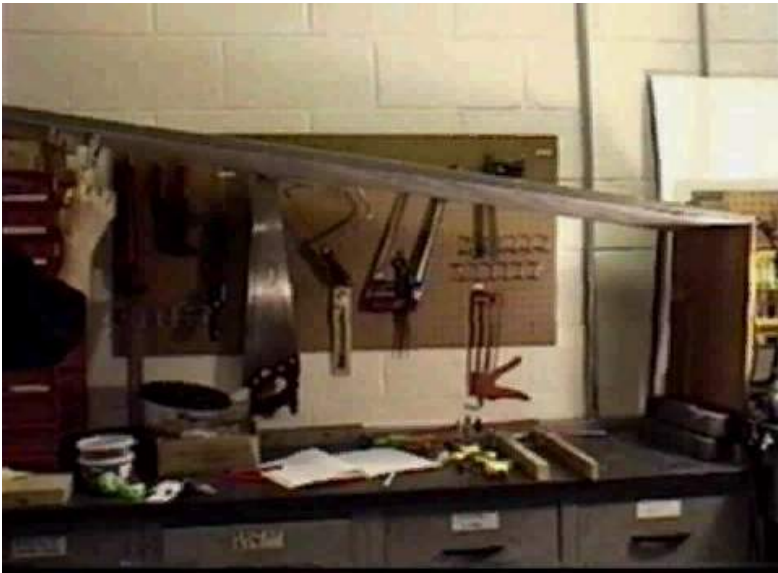
Fuminori Saito, Toshio Fukuda, and Fumihito Arai







Coleman and Ruina



**Marc H. Raibert**  
**H. Benjamin Brown, Jr.**  
**Michael Chepponis**

Department of Computer Science  
and The Robotics Institute  
Carnegie-Mellon University  
Pittsburgh, Pennsylvania 15213

### Abstract

*In order to explore the balance in legged locomotion, we are studying systems that hop and run on one springy leg. Previous work has shown that relatively simple algorithms can achieve balance on one leg for the special case of a system that is constrained mechanically to operate in a plane (Raibert, in press; Raibert and Brown, in press). Here we generalize the approach to a three-dimensional (3D) one-legged machine that runs and balances on an open floor without physical support. We decompose control of the machine into three separate parts: one part that controls forward running velocity, one part that controls attitude of the body, and a third part that controls hopping height. Experiments with a physical 3D one-legged hopping machine showed that this control scheme, while simple to implement, is powerful enough to permit hopping in place, running at a desired rate, and travel along a simple path. These algorithms that control locomotion in 3D are direct generalizations of those in 2D, with surprisingly little additional complication.*

### 1. Introduction

The ability to balance actively is a key ingredient in the mobility observed in natural legged systems and could be an important factor in human-made legged systems yet to be developed. Actively stabilized legged systems can move on a narrow base of support, permitting travel where obstacles are closely spaced or where the support path is narrow. Systems that balance need not be supported all the time and may therefore

This research was sponsored by a grant from the System Development Foundation, and by contract MDA903-81-C-0130 from the System Sciences Office of the Defense Advanced Research Projects Agency.

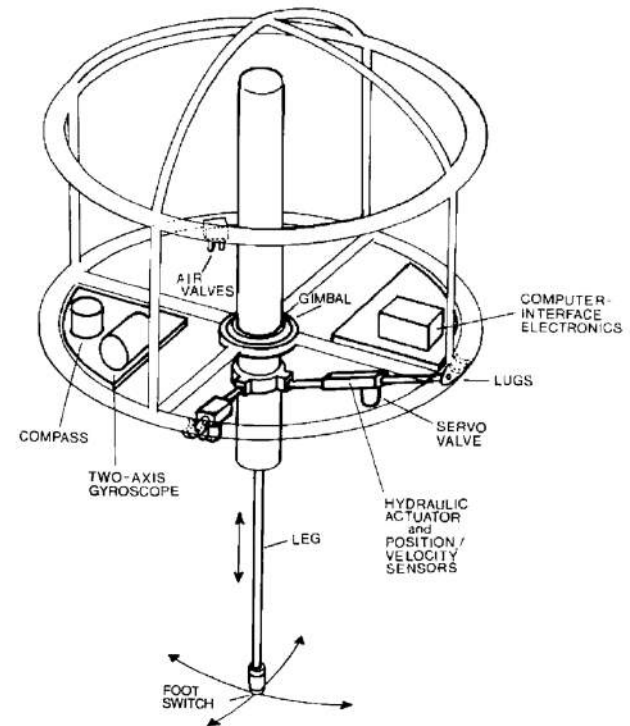
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Vol. 3, No. 2, Summer 1984,  
0278-3649/84/020075-18 \$05.00/0,  
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## Experiments in Balance with a 3D One-Legged Hopping Machine

use support points that are widely separated or erratically placed. This ability to place the feet on just those locations that provide good support increases the types of terrain a legged system can negotiate. Biological legged systems routinely operate with narrow base and intermittent support to traverse terrain too difficult for existing wheeled or tracked vehicles.

While the potential advantages of active stability and intermittent support may have been recognized for some time (Manter 1938; McGhee and Kuhner 1969; Frank 1970; Gubina 1972; Vukobratović 1973), progress in building legged systems that employ such principles has been retarded by the perceived difficulty of the task. As a result, much of the previous work on walking machines has taken a quasi-static approach, operating at low velocity with continuous and broad-based support (Frank 1968; Bessonov and Umnov 1973; McGhee and Buckett 1977; Hirose and Umetani 1980; Sutherland 1983). These devices have four or six legs, with at least three legs providing support at all times.

Our previous work has shown experimentally that it is possible to control a dynamic legged system that balances actively as it hops and runs (Raibert and Brown, 1984). However, the apparatus of those experiments was a planar device that was constrained mechanically to move with just three degrees of freedom. Useful locomotion takes place in 3D space, where motion with six degrees of freedom is possible. In this paper we present algorithms that control a legged system that balances as it hops and runs in 3D and experimental data that characterize the performance. These experiments show that, in the context of a hopping machine with a single springy leg, the control problem need not be difficult at all. A very simple set of algorithms is sufficient to control the machine as it hops in place, as it travels from point to point under velocity or position control, and as it responds to external mechanical disturbances. The control algorithms are direct generalizations of those used in 2D.



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School of Engineering and Applied Science  
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## An "Interesting" Strange Attractor in the Dynamics of a Hopping Robot

**Daniel E. Koditschek  
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Center for Systems Science  
Department of Electrical Engineering  
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New Haven, Connecticut 06520-1968

## Analysis of a Simplified Hopping Robot

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Ann Arbor, Michigan 48109

**P. J. Kindlmann**

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Department of Electrical Engineering  
Yale University  
New Haven, Connecticut 06520-1968

## Planning and Control of Robotic Juggling and Catching Tasks



parkour

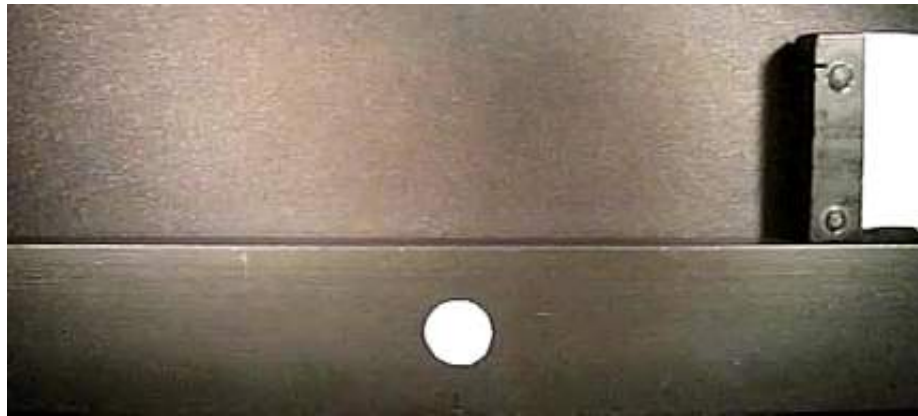


the ParkourBot

Brown, Degani, Feng, Long, Choset, Mason, Lynch



# more limit cycles



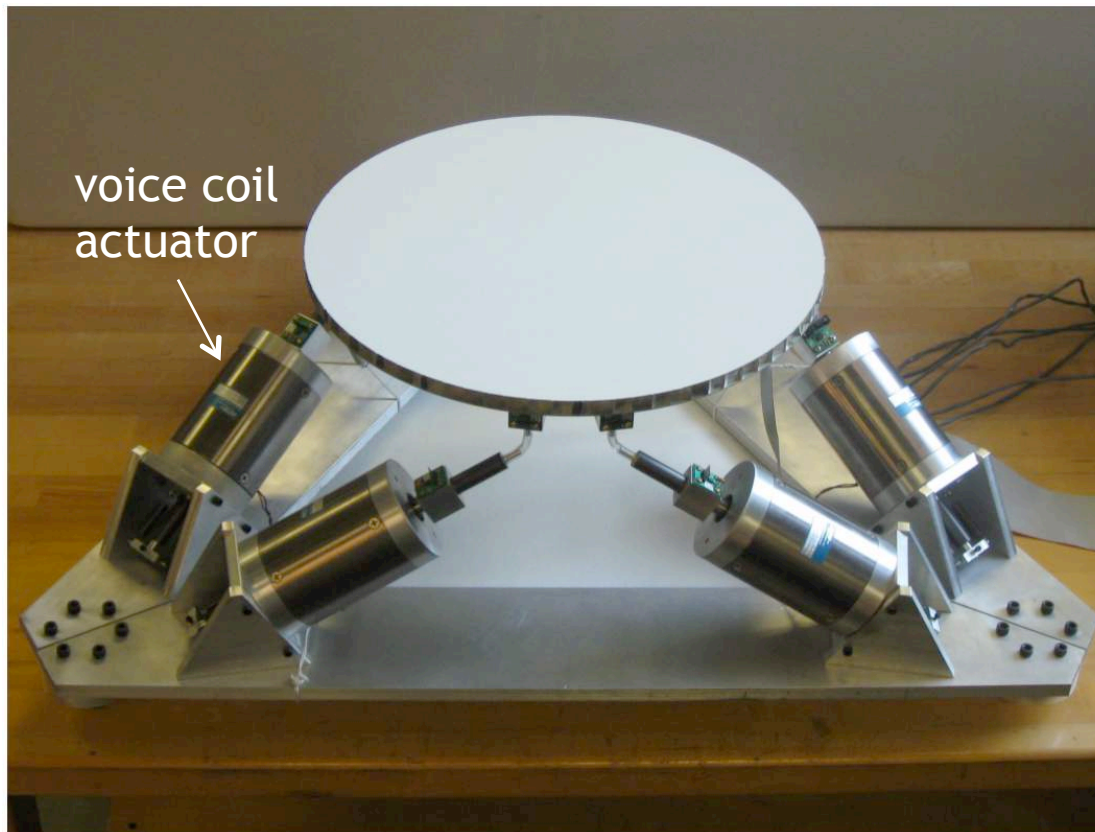
15 Hz vibration  
1/20 speed

$$f_{\text{fric}} = \mu f_{\text{normal}} \frac{\mathbf{v}_{\text{rel}}}{\|\mathbf{v}_{\text{rel}}\|}$$



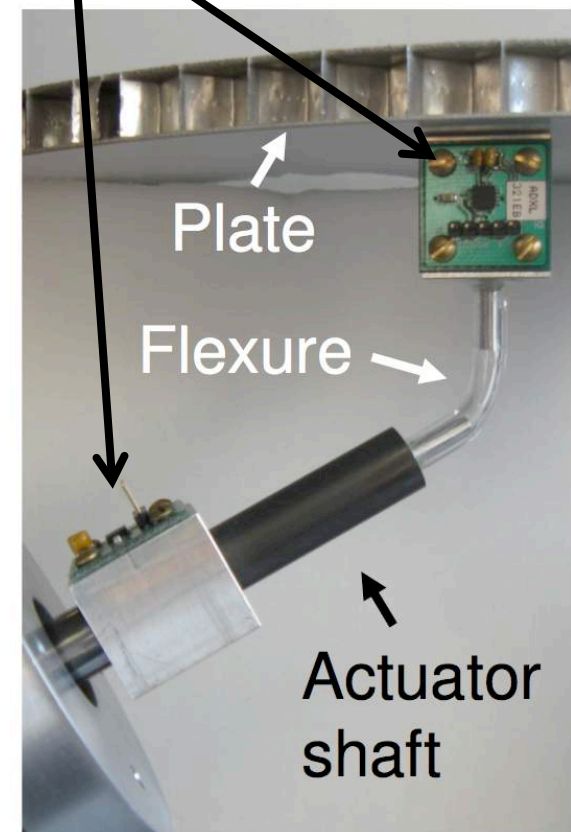
# the 6-dof PPOD

(Programmable Parts Orienting Device)



flexure-based Stewart platform

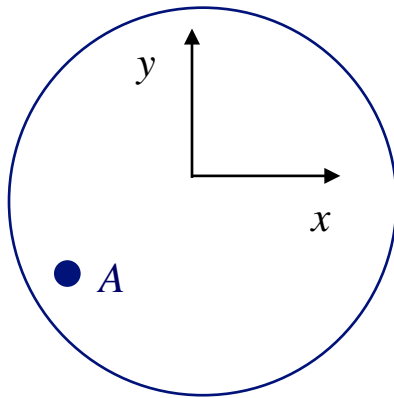
accelerometers



audio  
amps



# limit cycles of pursuer-evader



top view of plate

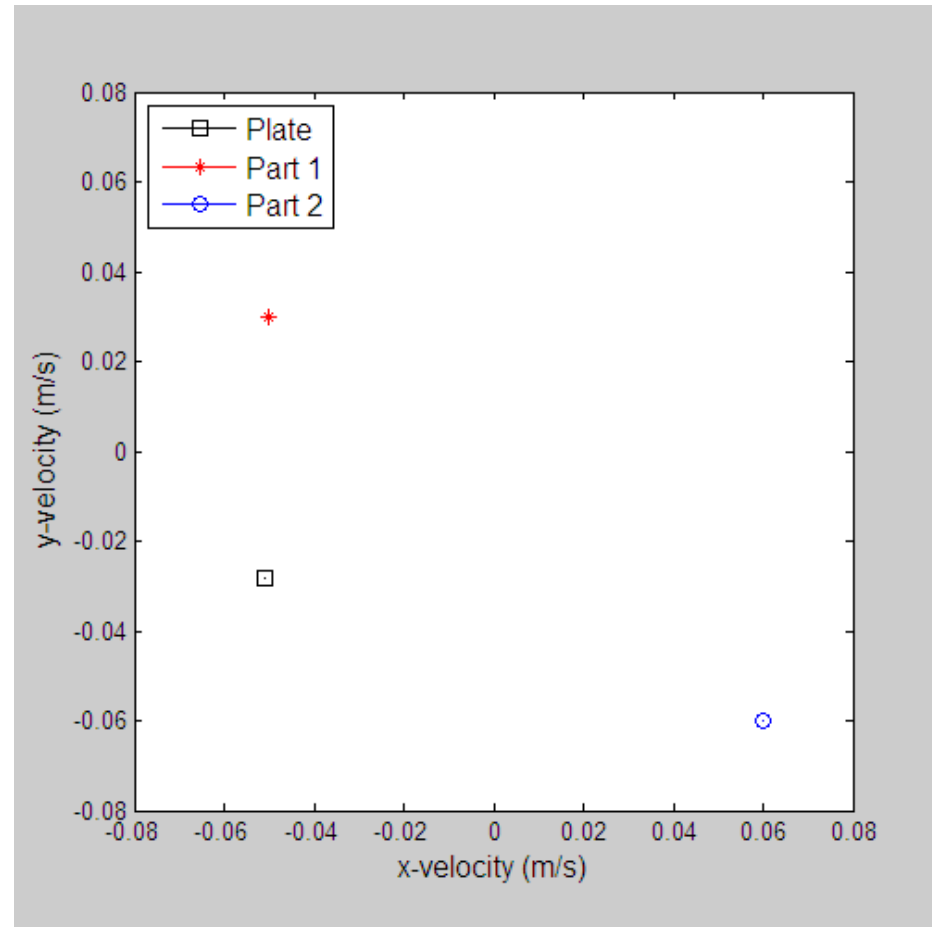
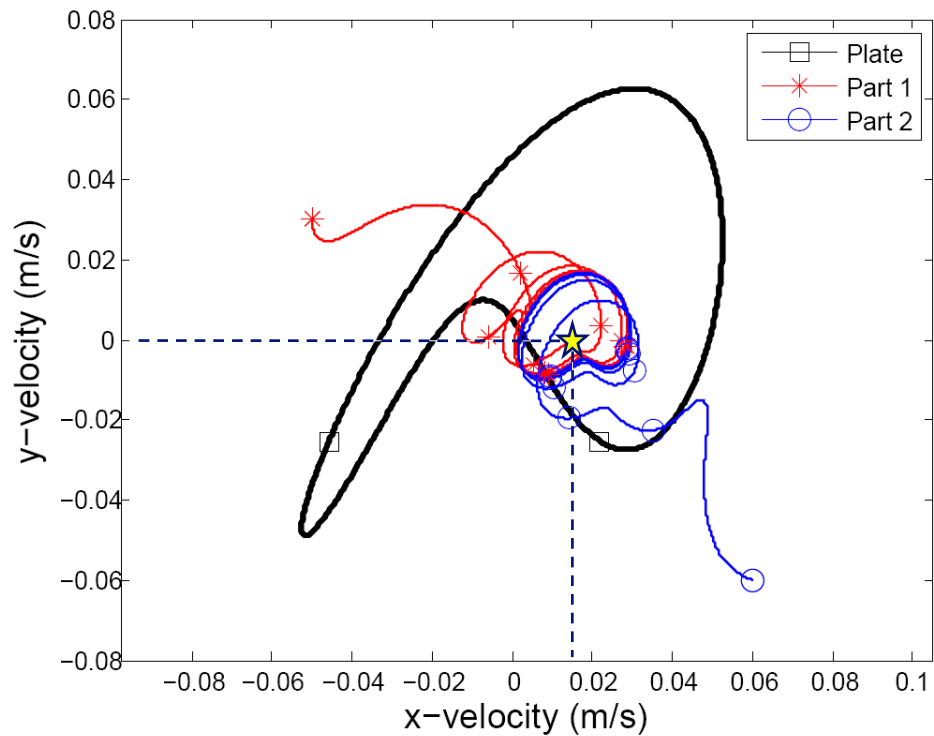


plate and part horizontal velocities at A

$$f_{\text{fric}} = \mu f_{\text{normal}} \frac{\mathbf{v}_{\text{rel}}}{\|\mathbf{v}_{\text{rel}}\|}$$

# asymptotic velocity

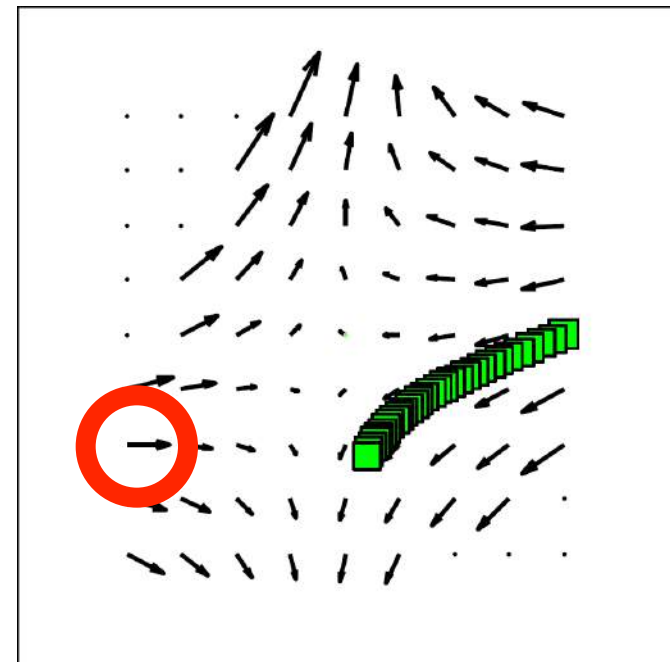
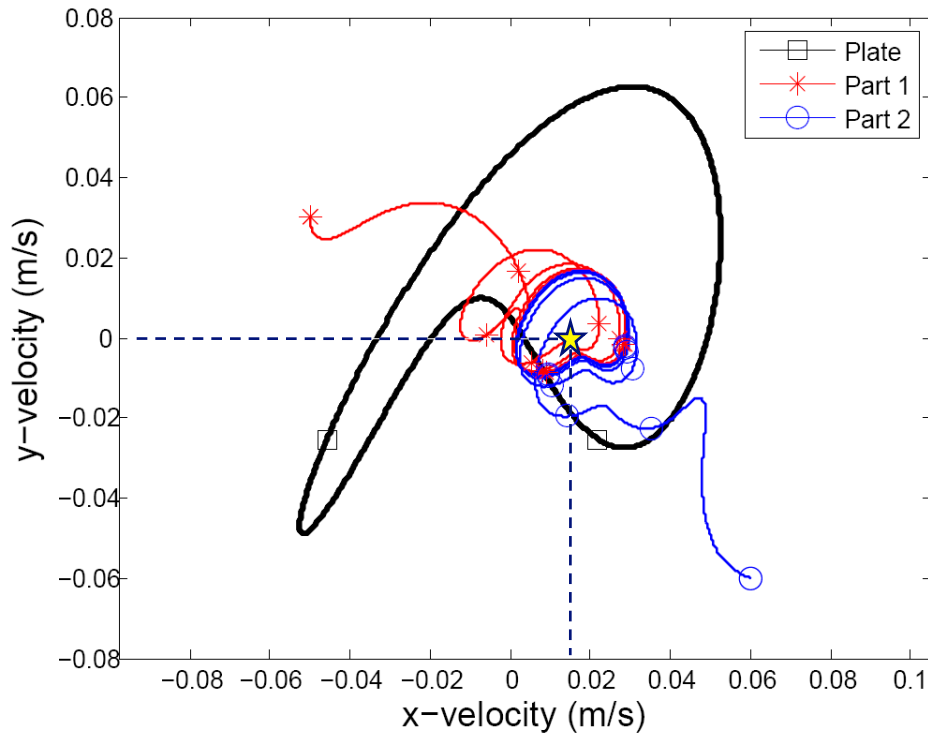


*Asymptotic velocity* at  $(x,y)$ :

$$\mathbf{v}(x, y) = \frac{1}{T} \int_0^T \mathbf{v}'(t) dt$$

where  $\mathbf{v}'(t)$  is the limit cycle.

# asymptotic velocity



*Asymptotic velocity* at  $(x, y)$ :

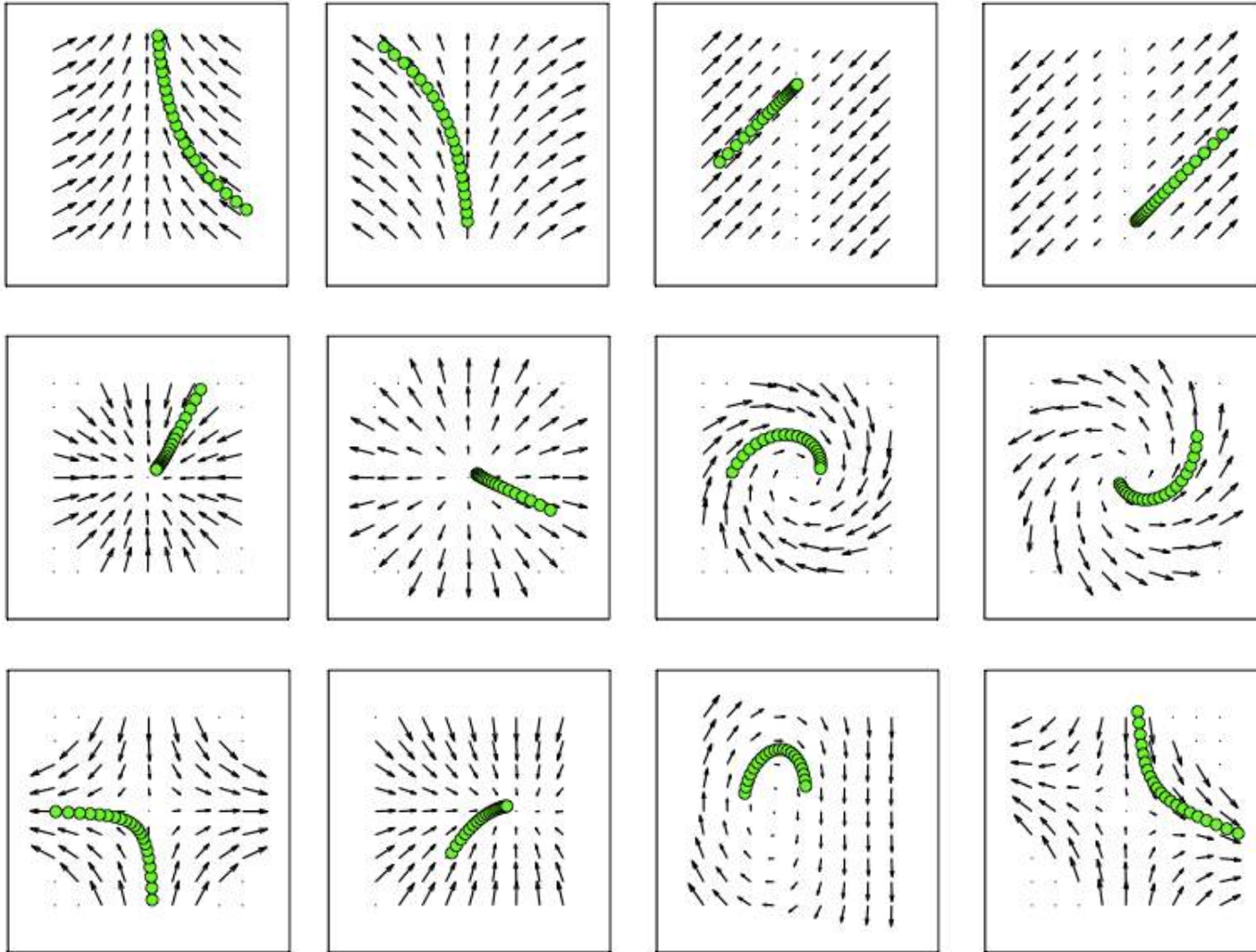
$$\mathbf{v}(x, y) = \frac{1}{T} \int_0^T \mathbf{v}'(t) dt$$

where  $\mathbf{v}'(t)$  is the limit cycle.

asymptotic velocity vectors at all points:

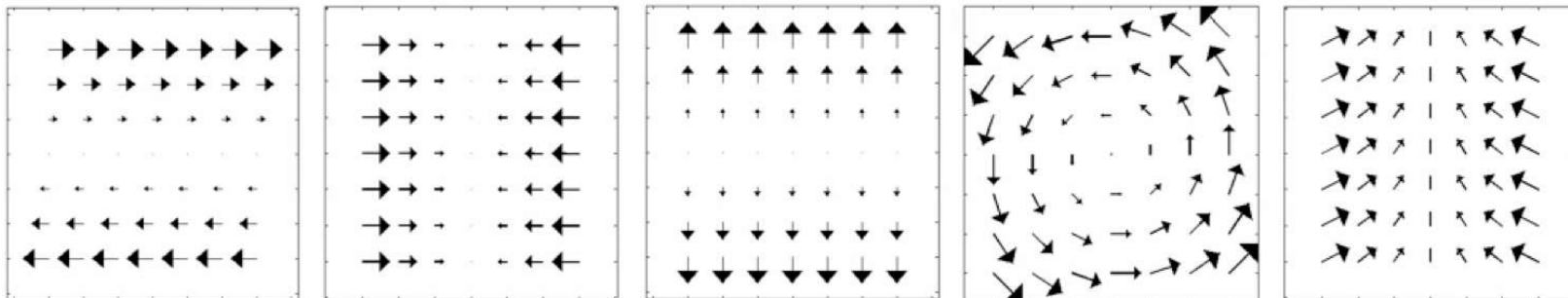
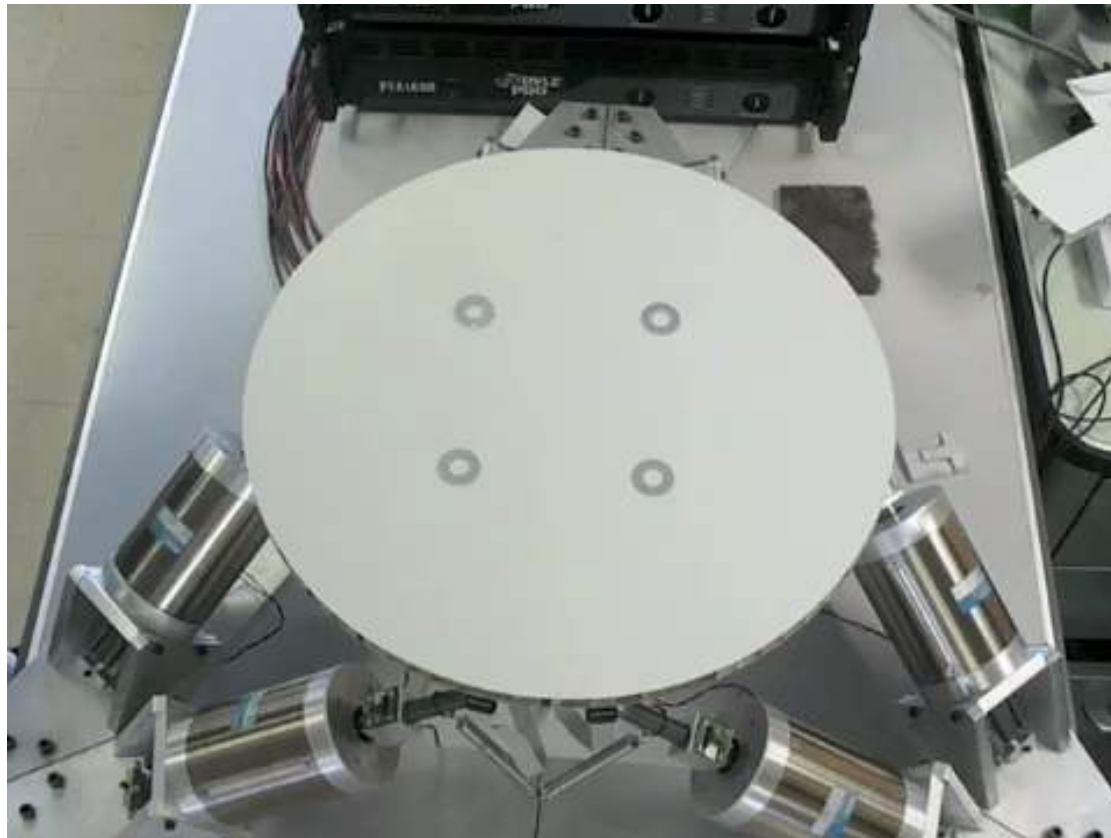
*asymptotic velocity field*

# asymptotic velocity fields

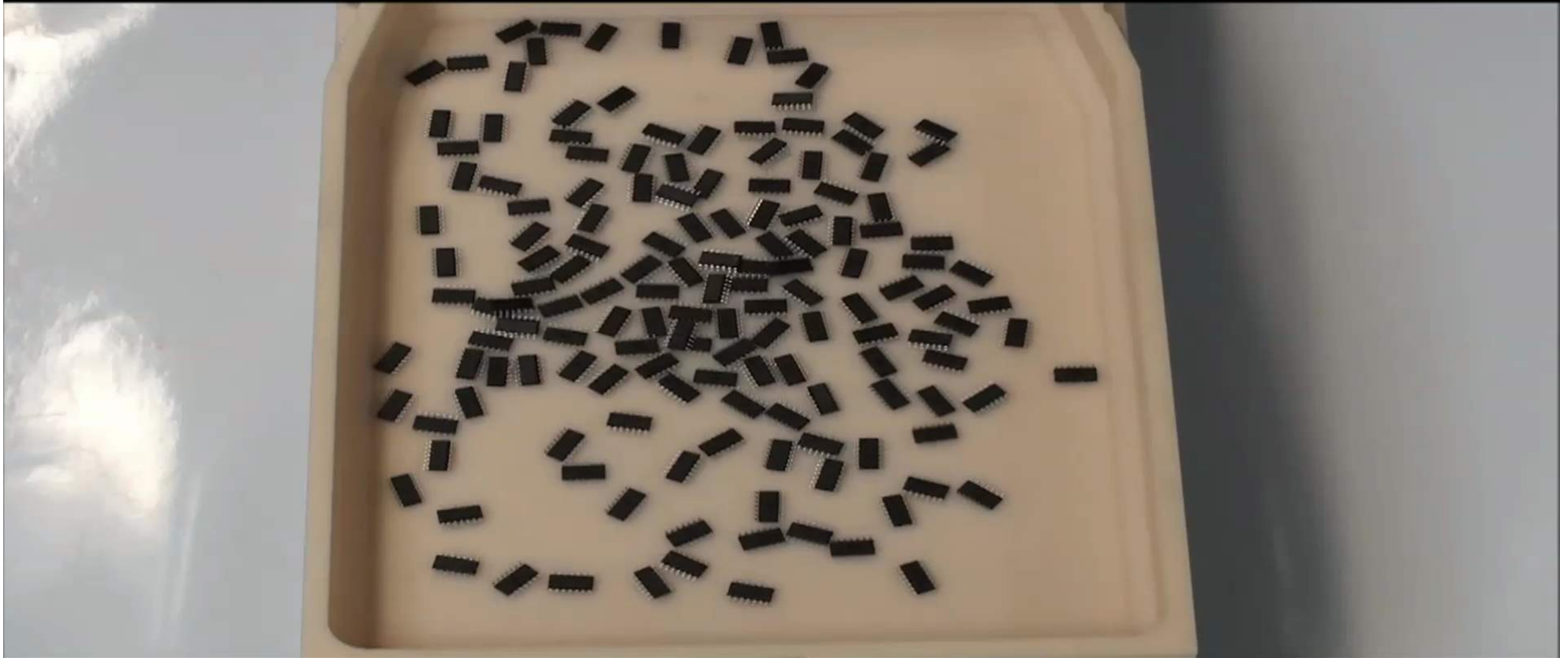


# the 6-dof PPOD

(Programmable Parts Orienting Device)



enter the "real world..."

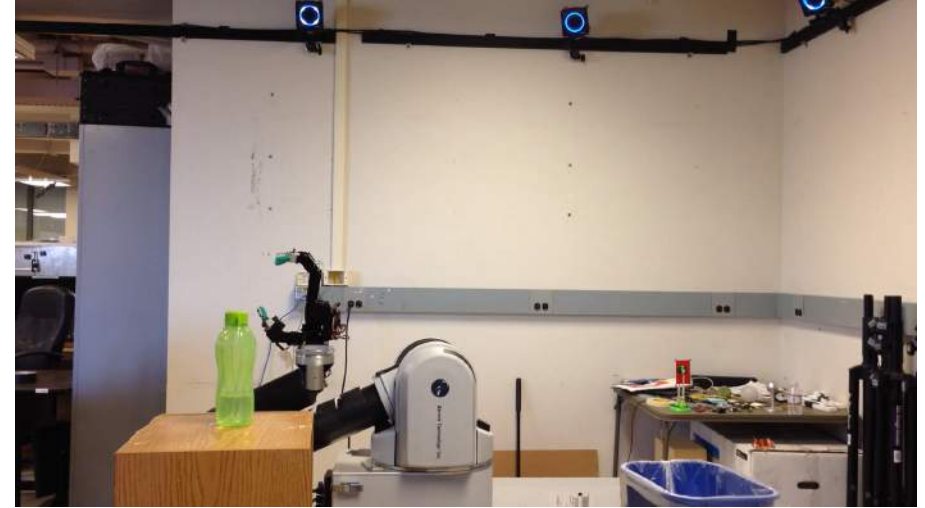
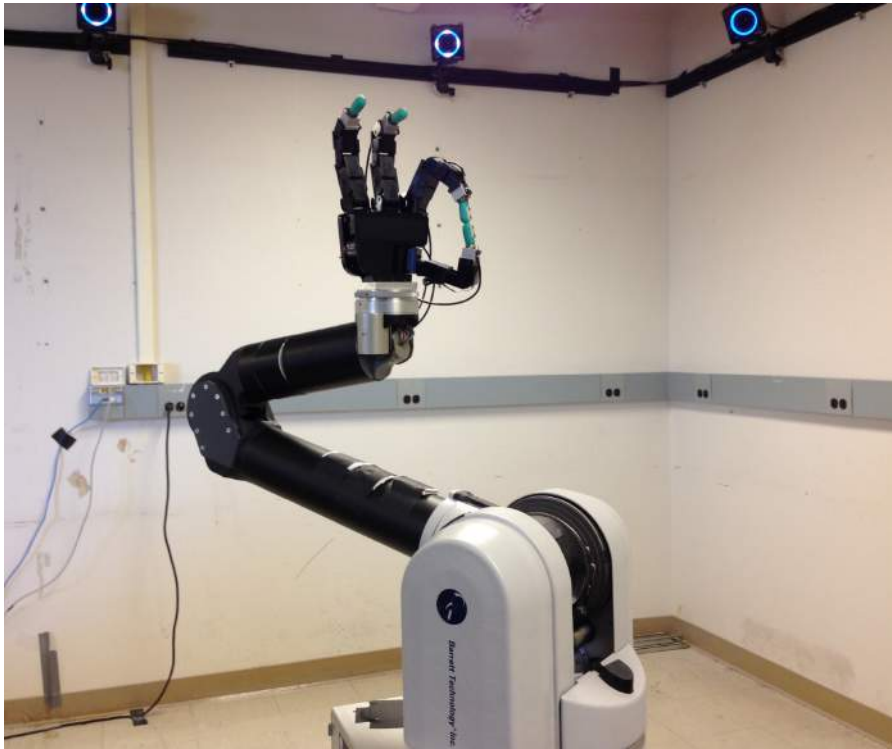


contact modeling?

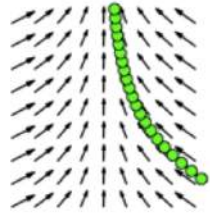
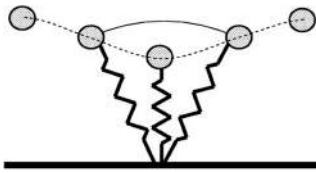




contact modeling?



ERIN  
7-dof WAM, 16-dof Allegro hand, 4 SynTouch BioTac fingertips, 10-camera OptiTrack



# the great divide

locomotion and manipulation



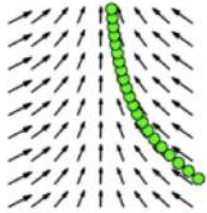
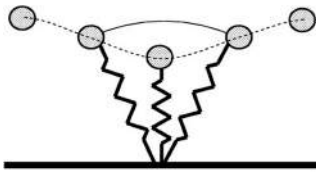
good low-level models  
kinematics,  $f = ma$ ,  
friction, impact



state estimation  
controllability  
motion planning  
feedback control

low-level models  
not available

- nonlinear
- underactuated
- nonholonomic
- hybrid
- obstacles
- optimality
- ...



# the great divide

locomotion and manipulation



good low-level models  
kinematics,  $f = ma$ ,  
friction, impact

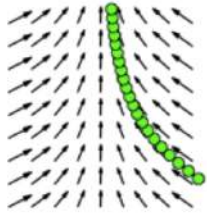
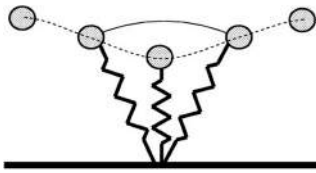


state estimation  
controllability  
motion planning  
feedback control

low-level models  
not available



qualitative physics  
machine learning  
statistical methods  
behaviors, heuristics  
passive mechanics



# the great divide

locomotion and manipulation



**templates**



good low-level models  
kinematics,  $f = ma$ ,  
friction, impact

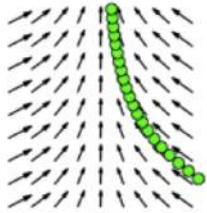
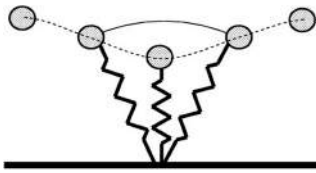


state estimation  
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# the great divide

locomotion and manipulation



good low-level models  
kinematics,  $f = ma$ ,  
friction, impact



state estimation  
controllability  
motion planning  
feedback control



## the future

- .....
- better sensors
  - better models
  - better simulators, faster computers
  - belief filtering for hybrid systems
  - belief controllability for hybrid systems
  - planning and control through multiple contact modes

low-level models  
not available



qualitative physics  
machine learning  
statistical methods  
behaviors, heuristics  
passive mechanics

